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FIXED POINT RESULTS FOR PAIRS OF ABSORBING MAPPINGS IN PARTIAL METRIC SPACES

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Abstract. The purpose of this paper is to extend and generalize Theorem 19 [19] for partial metric spaces using a new type of limit range property.

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1. Introduction

Let (X, d) be a metric space and S, T be two self mappings of X. Jungck [14] defined S and T to be compatible if

$$\lim_{n\to\infty} d(STx_n, TSx_n) = 0$$

whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$$

for some $t \in X$.

This concept has been frequently used to prove existence theorems in fixed point theory.

Let f, g be self maps of a nonempty set X. A point $x \in X$ is a coincidence point of f and g if $fx = gx$. The set of all coincidence points of f and g is denoted by $\mathcal{C}(f, g)$.

In 1994, Pant [20] introduced the notion of pointwise R - weakly commuting mappings in metric spaces, which is equivalent to commutativity in coincidence points.

Jungck [15] defined f and g to be weakly compatible if $fx = gx$ implies $fgx = gfx$. Thus, in metric spaces, f and g are weakly compatible if and only if f and g are pointwise R - weakly commuting.

The study of common fixed points for noncompatible mappings is also interesting. The work in this regard has been initiated by Pant [21], [22], [23].

Aamri and El Moutawakil [1] introduced a generalization of noncompatible mappings.

Definition 1 ([1])**.** *Let S and T be two self mappings of a metric space (X, d). We say that S and T satisfy $(E.A)$ - property if there exists a sequence $\{x_n\}$ in X such that*

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$$

for some $t \in X$.

Remark 1. *It is clear that two self mappings S and T of a metric space (X, d) will be noncompatible if there exists $\{x_n\}$ in X such that*

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$$

for some $t \in X$ but $\lim_{n\to\infty} d(STx_n, TSx_n)$ is nonzero or nonexistent. Therefore, two noncompatible self mappings of a metric space (X, d) satisfy $(E.A)$ - property.

It is known from [24] that the notions of weakly compatible mappings and mappings satisfying $(E.A)$ - property are independent.

Liu et al. [17] defined the notion of common $(E.A)$ - property.

Definition 2 ([17])**.** *Two pairs (A, S) and (B, T) of self mappings on a metric space (X, d) are said to satisfy common $(E.A)$ - property if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that*

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Ty_n = \lim_{n\to\infty} By_n = t$$

for some $t \in X$.

There exists a vast literature concerning the study of fixed points for mappings satisfying $(E.A)$ - property.

In 2011, Sintunavarat and Kumam [31] introduced the notion of common limit range property.

Definition 3 ([31])**.** *A pair (A, S) of self mappings of a metric space (X, d) is said to satisfy the common limit range property with respect to S, denoted $CLR_{(S)}$ - property, if there exists a sequence $\{x_n\}$ in X such that*

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$$

for some $t \in S(X)$.

Thus we can infer that a pair (A, S) satisfying $(E.A)$ - property along with the closedness of the subspace $S(X)$ always have $CLR_{(S)}$ - property with respect to S.

Recently, Imdad et al. [11] introduced the notion of common limit range property for two pairs of self mappings.

Definition 4 ([11]). *Two pairs (A, S) and (B, T) of self mappings in a metric space (X, d) are said to satisfy common limit range property with respect to S and T, denoted $CLR_{(S,T)}$ - property, if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that*

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = t$$

for some $t \in S(X) \cap T(X)$.

Some results for pairs of mappings satisfying $CLR_{(S)}$ and $CLR_{(S,T)}$ - property are obtained in [12], [10], [13].

A new type of limit range property is introduced in [27].

Definition 5 ([27]). *Let A, S, T be self mappings of a metric space (X, d). The pair (A, S) is said to satisfy common limit range property with respect to T, denoted $CLR_{(A,S)T}$ - property, if there exists a sequence $\{x_n\}$ in X such that*

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$$

for some $t \in S(X) \cap T(X)$.

Example 1. *Let $\mathbb{R}_+$ be the metric space with the usual metric space, $Ax = \frac{x^2+1}{2}, Sx = \frac{x+1}{2}, Tx = x + \frac{1}{4}$. Then $S(X) = \left[\frac{1}{2}, \infty\right)$, $T(X) = \left[\frac{1}{4}, \infty\right)$, $S(X) \cap T(X) = \left[\frac{1}{2}, \infty\right)$. Let $\{x_n\}$ be a sequence in X such that $\lim_{n\to\infty} x_n = 0$. Then $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \frac{1}{2} \in S(X) \cap T(X)$.*

Remark 2. *Let A, B, S and T satisfy the common limit range property with respect to (S,T). Then (A, S) satisfy the common limit range property with respect to T. The converse is not true. If $B = x^2 + \frac{1}{4}$, then $\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = \frac{1}{4} \neq \frac{1}{2}$. Hence, the pairs (A, S) and (B, T) don't satisfy $CLR_{(S,T)}$ - property.*

2. Preliminaries

In 1994, Matthews [18] introduced the concept of partial metric space as a part of the study of denotional semantics of dataflow networks and proved the Banach contraction principle in such spaces. Many authors studied fixed points for mappings satisfying contractive conditions in complete partial metric spaces.

Recently, in [2], [4], [6], [16] and in other papers some fixed point theorems under various contractive conditions are proved.

Definition 6 ([18]). *Let X be a nonempty set. A function $p : X \times X \to \mathbb{R}_+$ is said to be a partial metric on X if for all $x, y, z \in X$, the following conditions hold:*

$(P_1) : p(x,x) = p(y,y) = p(x,y)$ *if and only if* $x = y$,

$(P_2) : p(x,x) \leq p(x,y)$,

$(P_3) : p(x,y) = p(y,x)$,

$(P_4) : p(x,z) \leq p(x,y) + p(y,z) - p(y,y)$.

The pair (X,p) is called a partial metric space.

If $p(x,y) = 0$, then $x = y$, but the converse does not always hold.

Each partial metric p on X generates a T_0 - topology τ_p which has as base the family of p - open balls $\{B_p(x,\varepsilon) : x \in X,\ \varepsilon > 0\}$, where $B_p(x,\varepsilon) = \{y \in X : p(x,y) \leq p(x,x) + \varepsilon\}$ for all $x \in X$ and $\varepsilon > 0$.

A sequence $\{x_n\}$ of a partial metric space (X,p) converges to a point $x \in X$ with respect to τ_p, denoted $x_n \to x$, if and only if $p(x,x) = \lim_{n\to\infty} p(x_n,x)$.

If p is a partial metric on X, then the function

$$d_p(x,y) = 2p(x,y) - p(x,x) - p(y,y)$$

defines a metric on X.

Further, a sequence $\{x_n\}$ in (X, d_p) converges to a point $x \in X$ if

$$\lim_{n,m\to\infty} p(x_n,x_m) = \lim_{n\to\infty} p(x_n,x) = p(x,x). \tag{1}$$

Lemma 1 ([2], [18]). *Let (X,p) be a partial metric space and $\{x_n\}$ is a sequence in X which converges to a point z with $p(z,z) = 0$. Then*

$$\lim_{n\to\infty} p(x_n,y) = p(z,y)$$

for every $y \in X$.

Definition 7 ([18]). *Let (X,p) be a partial metric space. A sequence $\{x_n\}$ in X is a Cauchy sequence in (X,p) if and only if $\lim_{n,m\to\infty} p(x_n,x_m)$ exists and is finite.*

The notion of common limit range property for a pair of mappings in partial metric space is defined in [28].

Definition 8 ([28]). *A pair (A,S) of self mappings of a partial metric space (X,p) is said to satisfy the limit range property with respect to S, denoted $CLR_{(S)}$ - property, if there exists a sequence $\{x_n\}$ in X such that*

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$$

for some $t \in S(X)$ and $p(x,x) = 0$.

Definition 9. *Let A, S, T be three self mappings of a partial metric space (X, p). Then (A, S) and T satisfy common limit range property with respect to T, if there exists a sequence $\{x_n\}$ in X such that*

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$$

for some $t \in S(X) \cap T(X)$ and $p(x,x) = 0$.

Example 2. *Let $X = [0,4]$ be a partial metric space with*

$$p(x,y) = \begin{cases} |x-y|, & x,y \in [0,2] \\ \max\{x,y\}, & x,y \in (2,4] \end{cases}$$

and $Ax = \begin{cases} 2-x, & x \in [0,2] \\ \frac{2-x}{3}, & x \in (2,4] \end{cases}$, $Sx = \begin{cases} \frac{3-x}{2}, & x \in [0,2] \\ \frac{x}{2}, & x \in (2,4] \end{cases}$, $Tx = x$. *For an increasing sequence $\{x_n\}$ in X such that $x_n \to 1$, then $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t = 1$. Obviously, $S(X) \cap T(X) = S(X)$ and $t \in S(X)$ with $p(t,t) = p(1,1) = 0$.*

Definition 10 ([7], [30])**.** *Let A and S be two self mappings of a metric space (X,d). A is said to be S - absorbing if there exists $R > 0$ such that*

$$d(Sx, SAx) \leq Rd(Sx, Ax)$$

for all $x \in X$.

Similarly, S is said to be A - absorbing if there exists $R > 0$ such that

$$d(Ax, ASx) \leq Rd(Ax, Sx)$$

for all $x \in X$.

Definition 11 ([7], [9])**.** *A is said to be pointwise S - absorbing if for given $x \in X$, there exists $R > 0$ such that*

$$d(Sx, SAx) \leq Rd(Sx, Ax).$$

S is said to be pointwise A - absorbing if for given $x \in X$, there exists $R > 0$ such that

$$d(Ax, ASx) \leq Rd(Ax, Sx).$$

Remark 3. *If (X,p) is a partial metric space we have similar definitions of Definitions 10 and 11 with p instead of d.*

3. Implicit relations

Several fixed point theorems and common fixed point theorems have been unified considering a general condition by an implicit function [25], [26]. Recently, the method is used in the study of fixed points in metric spaces, symmetric spaces, quasi - metric spaces, b - metric spaces, ultra metric spaces, convex metric spaces, Hilbert spaces, compact metric spaces, for single - valued mappings, hybrid pairs of mappings and set - valued mappings.

Also, this method is used in the study of fixed points for mappings satisfying a contractive/extensive condition of integral type, in fuzzy metric spaces, probabilistic metric spaces, intuitionistic metric spaces, G - metric spaces, G_p - metric spaces. With this method, the proofs of some fixed point theorems are more simple. As well, the method allows the study of local and global properties of fixed point structures.

Some fixed point theorems for mappings satisfying implicit relations in partial metric spaces are proved in [5], [8], [9], [28] - [31].

In 2008, Ali and Imdad [3] introduced a new class of implicit relations.

Definition 12 ([3]). *Let $\mathcal{F}$ be the family of lower semi - continuous functions $F : \mathbb{R}_+^6 \to \mathbb{R}$ satisfying the following conditions:*

$(F_1) : F(t,0,t,0,0,t) > 0$, $\forall t > 0$,

$(F_2) : F(t,0,0,t,t,0) > 0$, $\forall t > 0$,

$(F_3) : F(t,t,0,0,t,t) > 0$, $\forall t > 0$.

Example 3. $F(t_1,...,t_6) = t_1 - k\max\{t_2,t_3,...,t_6\}$, *where* $k \in [0,1)$.

Example 4. $F(t_1,...,t_6) = t_1 - k\max\left\{t_2,t_3,t_4,\dfrac{t_5+t_6}{2}\right\}$, *where* $k \in [0,1)$.

Example 5. $F(t_1,...,t_6) = t_1 - k\max\left\{t_2,\dfrac{t_3+t_4}{2},\dfrac{t_5+t_6}{2}\right\}$, *where* $k \in [0,1)$.

Example 6. $F(t_1,...,t_6) = t_1 - at_2 - bt_3 - ct_4 - dt_5 - et_6$, *where* $a,b,c,d,e \geq 0$, $a+d<1$ *and* $c+d<1$.

Example 7. $F(t_1,...,t_6) = t_1 - \alpha\max\{t_2,t_3,t_4\} - (1-\alpha)(at_5+bt_6)$, *where* $\alpha \in (0,1)$, $a,b \geq 0$ *and* $a+b \leq 1$.

Example 8. $F(t_1,...,t_6) = t_1 - at_2 - \dfrac{b(t_5+t_6)}{1+t_3+t_4}$, *where* $a,b \geq 0$ *and* $a+2b<1$.

Example 9. $F(t_1,...,t_6) = t_1 - \max\{ct_2,ct_3,ct_4,at_5+bt_6\}$, *where* $c \in (0,1)$, $a,b \geq 0$ *and* $a+2b<1$.

Example 10. $F(t_1,...,t_6) = t_1 - at_2 - b\sqrt{t_3t_4} - c\sqrt{t_5t_6}$, *where* $a,b \geq 0$ *and* $a+c<1$.

The following theorem is proved in [19].

Theorem 2. *Let A, B, S and T be self mappings of a metric space (X, d) satisfying:*

1) $S(X)$ and $T(X)$ are closed subsets of X,

2) the pairs (A, S) and (B, T) enjoy the common property $(E.A)$,

3) for all $x, y \in X$ and some $F \in \mathcal{F}$

$$F\left(\begin{array}{c} d(Ax, By), d(Sx, Ty), d(Sx, Ax), \\ d(Ty, By), d(Sx, By), d(Ax, Ty) \end{array}\right) \leq 0.$$

Then the pairs (A, S) and (B, T) have a coincidence point. Moreover, if A is pointwise S - absorbing and B is pointwise T - absorbing, then A, B, S and T have a unique common fixed point.

The purpose of this paper is to extend Theorem 2 for partial metric spaces using a new type of limit range property.

4. Main results

Theorem 3. *Let A, B, S and T be self mappings of a partial metric space (X, p) satisfying*

$$F\left(\begin{array}{c} p(Ax, By), p(Sx, Ty), p(Sx, Ax), \\ p(Ty, By), p(Sx, By), p(Ax, Ty) \end{array}\right) \leq 0 \tag{2}$$

for all $x, y \in X$ and some $F \in \mathcal{F}$.

Then A, B, S and T have at most one common fixed point z such that $p(z, z) = 0$.

Proof. Suppose that A, B, S and T have two common fixed points z_1, z_2 such that $p(z_i, z_i) = 0$ for $i = 1, 2$. Then by (2) we have

$$F\left(\begin{array}{c} p(Az_1, Bz_2), p(Sz_1, Tz_2), p(Sz_1, Az_1), \\ p(Tz_2, Bz_2), p(Sz_1, Bz_2), p(Az_1, Tz_2) \end{array}\right) \leq 0,$$

$$F(p(z_1, z_2), p(z_1, z_2), 0, 0, p(z_1, z_2), p(z_1, z_2)) \leq 0,$$

a contradiction of (F_3) if $p(z_1, z_2) > 0$. Hence $p(z_1, z_2) = 0$, which implies $z_1 = z_2$.

Theorem 4. *Let A, B, S and T be self mappings of a partial metric space (X, p) satisfying inequality (2) holds for all $x, y \in X$ and some $F \in \mathcal{F}$.*

If A, S, B and T satisfy $CLR_{(A,S)}$ - property, then

1) $\mathcal{C}(A, S) \neq \emptyset$,

2) $\mathcal{C}(B, T) \neq \emptyset$.

Moreover, if A is pointwise S - absorbing and B is pointwise T - absorbing, then A, B, S and T have a unique common fixed point z such that $p(z, z) = 0$.

Proof. Since (A,S) and T satisfy $CLR_{(A,S)T}$ - property, then there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z,$$

$z \in S(X) \cap T(X)$ and $p(z,z) = 0$.

Since $z \in T(X)$, there exists $u \in X$ such that $z = Tu$. By (2) we obtain

$$\begin{array}{c} F(p(Ax_n, Bu), p(Sx_n, Tu), p(Sx_n, Ax_n), \\ p(Tu, Bu), p(Sx_n, Bu), p(Ax_n, Tu)) \leq 0. \end{array} \tag{3}$$

By (P_4), $p(Sx_n, Ax_n) \leq p(Sx_n, z) + p(z, Ax_n)$.

Letting n tends to infinity, by Lemma 1 we obtain

$$\lim_{n\to\infty} p(Sx_n, Ax_n) = 0.$$

Letting $n \to \infty$ in (2), by Lemma 1 we obtain

$$F(p(z,Bu), 0, 0, p(z,Bu), p(z,Bu), 0) \leq 0,$$

a contradiction of (F_2) if $p(z,Bu) > 0$. Hence $p(z,Bu) = 0$ which implies $z = Bu = Tu$ and $\mathcal{C}(B,T) \neq \emptyset$.

Also, since $z \in S(X)$, there exists $v \in X$ such that $z = Sv$.

By (2) we obtain

$$\begin{array}{c} F(p(Av, Bu), p(Sv, Tu), p(Sv, Av), \\ p(Tu, Bu), p(Sv, Bu), p(Av, Tu)) \leq 0, \end{array}$$

$$F(p(Av, z), 0, p(z, Av), 0, 0, p(z, Av)) \leq 0,$$

a contradiction of (F_1) if $p(z,Av) > 0$. Hence $p(z,Av) = 0$ which implies $z = Av = Sv$, therefore $\mathcal{C}(A,S) \neq \emptyset$. Thus,

$$z = Av = Sv = Bu = Tu \text{ and } p(z,z) = 0.$$

Moreover, if A is pointwise S - absorbing, there exists $R_1 > 0$ such that

$$p(Sv, SAv) \leq R_1 p(Sv, Av) = R_1 p(z,z) = 0.$$

Hence $z = Sv = SAv = Sz$ and z is a fixed point of S.

Now we prove that $z = Az$. By (2) we have

$$F\left(\begin{array}{c} p(Az, Bu), p(Sz, Tu), p(Sz, Az), \\ p(Tu, Bu), p(Sz, Bu), p(Az, Tu) \end{array}\right) \leq 0,$$

$$F\left(p\left(Az,z\right),0,p\left(z,Az\right),0,0,p\left(Az,z\right)\right)\leq 0,$$

a contradiction of (F_1) if $p(z,Az)>0$. Hence $p(z,Az)=0$ which implies $z=Az$ and z is a common fixed point of A and S.

If B is pointwise T - absorbing, there exists $R_2>0$ such that

$$p\left(Tu,TBu\right)\leq R_2 p\left(Tu,Bu\right)=R_2 p\left(z,z\right)=0.$$

Hence $z=Tu=TBu=Tz$ and z is a fixed point of T.

By (2) we have

$$F\left(\begin{array}{c} p\left(Av,Bz\right),p\left(Sv,Tz\right),p\left(Sv,Av\right), \\ p\left(Tz,Bz\right),p\left(Sv,Bz\right),p\left(Av,Tz\right)\end{array}\right)\leq 0,$$

$$F\left(p\left(z,Bz\right),0,0,p\left(z,Bz\right),p\left(z,Bz\right),0\right)\leq 0,$$

a contradiction of (F_2) if $p(z,Bz)>0$. Hence $p(z,Bz)=0$ which implies $z=Bz$ and z is a common fixed point of B and T.

Then z is a common fixed point of A,B,S and T with $p\left(z,z\right)=0$.

By Theorem 3, z is the unique common fixed point of A,B,S and T with $p\left(z,z\right)=0$.

Example 11. *Let $X=[0,1]$ be a partial metric space with $p\left(x,y\right)=\max\{x,y\}$ and $Ax=0$, $Sx=\frac{x}{x+2}$, $Bx=\frac{x}{3}$ and $Tx=x$. So $A\left(X\right)=[0,1], S\left(X\right)=\left[0,\frac{1}{3}\right], T\left(X\right)=[0,1), S\left(X\right)\cap T\left(X\right)=\left[0,\frac{1}{3}\right]$. Then*

$$p\left(Sx,SAx\right)=p\left(\frac{x}{x+2},0\right)=\frac{x}{x+2},$$

$$p\left(Sx,Ax\right)=\frac{x}{x+2}.$$

Hence

$$p\left(Sx,SAx\right)\leq R_1 p\left(Sx,Ax\right)\ \text{for } R_1\geq 1.$$

$$p\left(Tx,TBx\right)=\max\left\{x,\frac{x}{3}\right\}=x,$$

$$p\left(Tx,Bx\right)=\max\left\{x,\frac{x}{3}\right\}=x.$$

Hence

$$p\left(Tx,TBx\right)\leq R_2 p\left(Tx,Bx\right)\ \text{for } R_2\geq 1.$$

So, A is pointwise S - absorbing and B is pointwise T - absorbing.

For an decreasing sequence $x_n\to 0$, then $\lim_{n\to\infty}Ax_n=\lim_{n\to\infty}Sx_n=0\in S\left(X\right)\cap T\left(X\right)$.

On the other hand,

$$p(Ax,By) = \max\left\{0, \frac{y}{3}\right\} = \frac{y}{3},$$

$$p(Ty,Ay) = \max\left\{y, \frac{y}{3}\right\} = y.$$

Hence,

$$p(Ax,By) \leq kp(Ty,By),$$

where $k \in \left[\frac{1}{3}, 1\right)$ *which implies*

$$\begin{aligned} p(Ax,By) &\leq k\max\{p(Sx,Ty), p(Sx,Ax), \\ &\quad p(Ty,By), p(Sx,By), p(Ax,Ty)\}, \end{aligned}$$

where $k \in \left[\frac{1}{3}, 1\right)$.

By Theorem 4, A, B, S *and* T *have a unique common fixed point* $z = 0$ *with* $p(0,0) = 0$.

Remark 4. *If* X *is a metric space we obtain a generalization of Theorem 2.*

5. Applications

5.1. Fixed points for mappings satisfying contractive conditions of integral type in partial metric space

For a function $f : (X,p) \to (X,p)$ we denote

$$pFix(f) = \{x \in X : x = fx \text{ and } p(x,x) = 0\}.$$

Theorem 5. *Let* A, B, S *and* T *be self mappings of a partial metric space* (X,p). *If inequality (2) holds for all* $x, y \in X$ *and some* $F \in \mathcal{F}$, *then we have*

$$[pFix(S) \cap pFix(T)] \cap pFix(A) = [pFix(S) \cap pFix(T)] \cap pFix(B).$$

Proof. Let $x \in [pFix(S) \cap pFix(T)] \cap pFix(A)$. Then by (2) we obtain

$$F\left(\begin{array}{c} p(Ax,Bx), p(Sx,Tx), p(Sx,Ax), \\ p(Tx,Bx), p(Sx,Bx), p(Ax,Tx) \end{array}\right) \leq 0,$$

$$F\left(\begin{array}{c} p(x,Bx), p(x,x), p(x,x), \\ p(x,Bx), p(x,Bx), p(x,x) \end{array}\right) \leq 0,$$

$$F(p(x,Bx), 0, 0, p(x,Bx), p(x,Bx), 0) \leq 0,$$

a contradiction of (F_2) if $p(x, Bx) > 0$. Hence $p(x, Bx) = 0$ which implies $x = Bx$. Therefore,

$$[pFix(S) \cap pFix(T)] \cap pFix(A) \subset [pFix(S) \cap pFix(T)] \cap pFix(B).$$

Similarly, by (2) and (F_1) we obtain

$$[pFix(S) \cap pFix(T)] \cap pFix(B) \subset [pFix(S) \cap pFix(T)] \cap pFix(A).$$

By Theorem 4 and Theorem 5 we obtain

Theorem 6. *Let S, T and $\{A_i\}_{i \in \mathbb{N}^*}$ be self mappings of a partial metric space (X, p) such that A_1, S and T satisfy $CLR_{(A_1,S)T}$ - property and*

$$F\left(\begin{array}{c} p(A_i x, A_{i+1} y), p(Sx, Ty), p(Sx, A_i x), \\ p(Ty, A_{i+1} y), p(Sx, A_{i+1} y), p(A_i x, Ty) \end{array}\right) \leq 0$$

for all $x, y \in X$, $i \in \mathbb{N}^*$ and some $F \in \mathcal{F}$.

If A_1 is pointwise S - absorbing and A_2 is pointwise T - absorbing, then S, T and $\{A_i\}_{i \in \mathbb{N}^*}$ have a unique common fixed point.

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