The Kobayashi Metric of a Complex Ellipsoid in $\mathbb{C}^2$

Brian E. Blank
Dashan Fan
David Klein
Steven G. Krantz
Daowei Ma
Myung-Yull Pang

Abstract

The infinitesimal Kobayashi metric of an ellipsoid of the form

$$E_m = \{(z_1, z_2) \in \mathbb{C}^2 : |z_1|^2 + |z_2|^{2m} < 1\}$$

is calculated explicitly, modulo a parameter that is determined by solving a transcendental equation. Using this result, we show that the metric is C^1 on the tangent bundle away from the zero section. We also describe software that will calculate, using a Monte Carlo method, the infinitesimal Kobayashi metric on a domain of the form

$$\Omega_\rho = \{(z_1, z_2) \in \mathbb{C}^2 : \rho(z_1, z_2) < 0\},$$

where ρ is a real-valued polynomial. We compare results of computer calculations with those obtained from the explicit formula for the Kobayashi metric.