Fast Fourier Analysis for SL_2 over a Finite Field and Related Numerical Experiments

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Abstract

We study the complexity of performing Fourier analysis for the group $mathrmSL_2(F_q)$, where F_q is the finite field of q elements. Direct computation of a complete set of Fourier transforms for a complex-valued function f on $mathrmSL_2(F_q)$ requires q^6 operations. A similar bound holds for performing Fourier inversion. Here we show that for both operations this naive upper bound may be reduced to $O(q^4 \log q)$, where the implied constant is universal, depending only on the complexity of the "classical" fast Fourier transform. The techniques we use depend strongly on explicit constructions of matrix representations of the group.

Additionally, the ability to construct the matrix representations permits certain numerical experiments. By quite general methods, recent work of others has shown that certain families of Cayley graphs on $mathrmSL_2(F_q)$ are expanders. However, little is known about their exact spectra. Computation of the relevant Fourier transform permits extensive numerical investigations of the spectra of these Cayley graphs. We explain the associated calculation and include illustrative figures.