# SOLVING THE QUINTIC BY ITERATION IN THREE DIMENSIONS

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ABSTRACT. The requirement for solving a polynomial is a means of breaking its symmetry, which in the case of the quintic, is that of the symmetric group $\mathcal{S}_5$. Induced by its five-dimensional linear permutation representation is a three-dimensional projective action. A mapping of complex projective 3-space with this $\mathcal{S}_5$ symmetry can provide the requisite symmetry-breaking tool.

The article describes some of the $\mathcal{S}_5$ geometry in $\mathbb{CP}^3$ as well as several maps with particularly elegant geometric and dynamical properties. Using a rational map in degree six, it culminates with an explicit algorithm for solving a general quintic. In contrast to the Doyle-McMullen procedure, which involves three 1-dimensional iterations, the present solution employs one 3-dimensional iteration.