INTEGRAL POINTS ON ELLIPTIC CURVES DEFINED BY SIMPLEST CUBIC FIELDS

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ABSTRACT. Let $f(X)$ be a cubic polynomial defining a simplest cubic field in the sense of Shanks. We study integral points on elliptic curves of the form $Y^2 = f(X)$. We compute the complete list of integral points on these curves for the values of the parameter below 1000. We prove that this list is exhaustive by using the methods of Tzanakis and de Weger, together with bounds on linear forms in elliptic logarithms due to S. David. Finally, we analyze this list and we prove in the general case the phenomena that we have observed. In particular, we find all integral points on the curve when the rank is equal to 1.