OFF-CENTER REFLECTIONS: CAUSTICS AND CHAOS

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ABSTRACT. We study the possible link between the dynamics of a certain family of circle maps and the caustics of their iterates. The maps are defined by *off-center reflections* in a mirrored circle; they can also be regarded as perturbed rotations. Some of our experimental observations can be justified rigorously: for example, a lower bound is given for the number of cusps and the mode-locking behavior are studied. Symplectic topology is a particularly useful tool in this study.