HEURISTICS ON TATE–SHAFAREVITCH GROUPS OF ELLIPTIC CURVES DEFINED OVER $\mathbb{Q}$

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ABSTRACT. In a well-known paper, Cohen and Lenstra gave conjectures on class groups of number fields. We give here similar conjectures for Tate–Shafarevitch groups of elliptic curves defined over $\mathbb{Q}$. For such groups (if they are finite), there exists a nondegenerate, alternating, bilinear pairing. We give some properties of such groups and then formulate heuristics which allow us to give precise conjectures.