# ON ARTIN $L$-FUNCTIONS FOR OCTIC QUATERNION FIELDS

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ABSTRACT. We study the Artin $L$-function $L(s,\chi)$ associated to the unique character $\chi$ of degree 2 in quaternion fields of degree 8. We first explain how to find examples of quaternion octic fields with not too large a discriminant. We then develop a method yielding a quick computation of the order $n_\chi$ of the zero of $L(s,\chi)$ at the point $s = \frac{1}{2}$. In all our calculations, we find that $n_\chi$ only depends on the sign of the root number $W(\chi)$; indeed $n_\chi = 0$ when $W(\chi) = +1$ and $n_\chi = 1$ when $W(\chi) = -1$. Finally we give some estimates on $n_\chi$ and low zeros of $L(s,\chi)$ on the critical line in terms of the Artin conductor $\mathfrak{f}_\chi$ of the character $\chi$.