THREE-MANIFOLDS HAVING COMPLEXITY AT MOST 9

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Abstract. We describe an algorithm which has enabled us to give a complete list, without repetitions, of all closed oriented irreducible three-manifolds of complexity up to 9. More interestingly, we have actually been able to give a name to each such manifold, by recognizing its canonical decomposition into Seifert fibered spaces and hyperbolic manifolds.

The algorithm relies on the extension of Matveev's theory of complexity to the case of manifolds bounded by suitably marked tori, and on the notion of assembling of two such manifolds. We show that every manifold is an assembling of manifolds which cannot be further disassembled, and we prove that there are surprisingly few such manifolds up to complexity 9.

Our most interesting experimental discovery is that there are 4 closed hyperbolic manifolds having complexity 9, and they are the 4 closed hyperbolic manifolds of least known volume. All other manifolds having complexity at most 9 are graph manifolds.