Asymptotic Behavior of Excitable Cellular Automata

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Abstract

We study two families of excitable cellular automata known as the *Greenberg–Hastings model* and the *cyclic cellular automaton.* Each family consists of local deterministic oscillating lattice dynamics, with parallel discrete-time updating, parametrized by the range ρ of interaction, l^p shape of its neighbor set, threshold θ for contact updating, and number κ of possible states per site. These models are mathematically tractable prototypes for the spatially distributed periodic wave activity of so-called excitable media observed in diverse disciplines of experimental science.

Fisch, Gravner and Griffeath [Fisch et al. 1991] studied experimentally the ergodic behavior of these models on $\mathbf{Z}^2$, started from random initial states. Among other phenomena, they noted the emergence of asymptotic phase diagrams (and dynamics on $\mathbf{R}^2$) in the threshold-range scaling limit as $\rho, \theta \to \infty$ with θ/ρ^2 constant.

Here we present several rigorous results and some experimental findings concerning various phase transitions in the asymptotic diagrams. Our efforts focus on evaluating bend(p), the limiting threshold cutoff for existence of the spirals that characterize many excitable media. Our main results are formulated in terms of spo(p), the cutoff for existence of *stable periodic objects* that arise as spiral cores. Some subtle consequences of anisotropic neighbor sets ($p \neq 2$) are also discussed; the case of box neighborhoods ($p = \infty$) is examined in detail.