Dynamics of Certain Nonconformal Degree-Two Maps of the Plane

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Abstract

We consider the rational maps given by $z \mapsto |z|^{2\alpha-2}z^2 + c$, for z and c complex and $\alpha > \frac{1}{2}$ fixed and real. The case $\alpha = 1$ corresponds to quadratic polynomials: some of the well-known results for this conformal case still hold for α near 1, while others break down. Among the differences between the two cases are the possibility, for $\alpha \neq 1$, of periodic attractors that do not attract the critical point, and the fact that for $\alpha < 1$ the Julia set is smooth for an open set of values of c. Numerical evidence suggests that the analogue of the Mandelbrot set for this family is connected, but not locally connected if $\alpha \neq 1$.