On Elliptic Diophantine Equations That Defy Thue's Method: The Case of the Ochoa Curve

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Abstract

The purpose of this paper is to show that elliptic diophantine equations cannot always be solved—in the most practical sense—by the Thue approach, that is, by solving each of the finitely many corresponding Thue equations of degree 4. After a brief general discussion, which is necessarily of a heuristic nature, to substantiate our claim, we consider the elliptic equation associated with the Ochoa curve. An explicit computational explanation as to the reasons for the failure of the Thue approach in this case is followed by a complete solution of the standard Weierstraß equation of this elliptic curve by a method which makes use of a recent lower bound for linear forms in elliptic logarithms.