# Periodicity and Transport from Round-Off Errors

Franco Vivaldi

**Abstract**

We investigate the effects of round-off errors on quasi-periodic motions in a linear symplectic planar map. By discretizing coordinates uniformly we transform this map into a permutation of $\mathbb{Z}^2$, and study motions near infinity, which correspond to a fine discretization. We provide numerical evidence that all orbits are periodic and that the average order of the period grows linearly with the amplitude. The discretization induces fluctuations of the invariant of the continuum system. We investigate the associated transport process for time scales shorter than the period, and we provide numerical evidence that the limiting behaviour is a random walk where the step size is modulated by a quasi-periodic function. For this stochastic process we compute the transport coefficients explicitly, by constructing their generating function. These results afford a probabilistic description of motions on a classical invariant torus.