Exceptional Units and Numbers of Small Mahler Measure

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Abstract

Let α be a unit of degree d in an algebraic number field, and assume that α is not a root of unity. We conduct a numerical investigation that suggests that if α has small Mahler measure, there are many values of n for which $1-\alpha^n$ is a unit and also many values of m for which $\Phi_m(\alpha)$ is a unit, where Φ_m is the m-th cyclotomic polynomial. We prove that the number of such values of n and m is bounded above by $O(d^{\,1+0.7/\log\log d})$, and we describe a construction of Boyd that gives a lower bound of $\Omega(d^{\,0.6/\log\log d})$.