On the Intersections of Longest Cycles in a Graph

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Abstract

We confirm a conjecture, due to Grötschel, regarding the intersection vertices of two longest cycles in a graph. In particular, we show that if G is a graph of circumference at least $k+1$, where $k \in \{6, 7\}$, and G has two longest cycles meeting in a set W of k vertices, then W is an articulation set. Grötschel had previously proved this result for $k \in \{3, 4, 5\}$ and shown that it fails for $k > 7$. As corollaries, we obtain results regarding the minimum lengths of longest cycles in certain vertex-transitive graphs. Our proofs are novel in that they make extensive use of a computer, although the programs themselves are straightforward.