Higher-Genus Chen–Gackstatter Surfaces and The Weierstrass Representation for Surfaces of Infinite Genus

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Abstract

Chen and Gackstatter [1982] constructed two complete minimal surfaces of finite total curvature, each having one Enneper-type end and all the symmetries of Enneper's surface. Karcher [1989] generalized the genus-one surface by increasing the winding order of the end. We prove that a similar generalization of the Chen–Gackstatter genus-two surface also exists. We describe a collection of immersed minimal surfaces that generalize both Chen–Gackstatter's and Karcher's surfaces by increasing the genus and the winding order of the end. The period problem associated with each of these surfaces is explained geometrically, and we present numerical evidence of its solvability for surfaces of genus as high as 35. We also make conjectures concerning these surfaces, and explain their motivation. Our numerical results led us to the Weierstrass data for several infinite-genus, one-ended, periodic minimal surfaces.