Phase Portraits of Planar Vector Fields: Computer Proofs

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Abstract

We present an algorithm for computer verification of the global structure of structurally stable planar vector fields. Constructing analytical proofs for the qualitative properties of phase portraits has been difficult. We try to avoid this barrier by augmenting numerical computations of trajectories of dynamical systems with error estimates that yield rigorous proofs. Our approach lends itself to high-precision estimates, because the proofs are broken into independent calculations whose length in floating-point operations does not increase with increasing precision. The algorithm is tested on a system that arises in the study of Hopf bifurcation of periodic orbits with 1:4 resonance.