Densities of Self-Similar Measures on the Line

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Abstract

We describe algorithms to compute self-similar measures associated to iterated function systems (i.f.s.) on an interval, and more general self-replicating measures that include Hausdorff measure on the attractor of a nonlinear i.f.s. We discuss a variety of error measurements for these algorithms. We then use the algorithms to study density properties of these measures experimentally. By *density* we mean the behavior of the ratio $\mu(B_r(x))/(2r)^\alpha$ as $r \to 0$, were α is an appropriate dimension. It is well-known that a limit usually does not exist. We have found an intriguing structure associated to these ratios that we call *density diagrams*. We also use density computations to approximate the exact Hausdorff measure of the attractor of an i.f.s.