An Investigation of Bounds for the Regulator of Quadratic Fields

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Abstract

It is well known that the nontorsion part of the unit group of a real quadratic field $\mathcal{K}$ is cyclic. With no loss of generality we may assume that it has a generator $\epsilon_0 > 1$, called the fundamental unit of $\mathcal{K}$. The natural logarithm of ϵ_0 is called the regulator R of $\mathcal{K}$. This paper considers the following problems: How large, and how small, can R get? And how often?

The answer is simple for the problem of how small R can be, but seems to be extremely difficult for the question of how large R can get. In order to investigate this, we conducted several large-scale numerical experiments, involving the Extended Riemann Hypothesis and the Cohen–Lenstra class number heuristics. These experiments provide numerical confirmation for what is currently believed about the magnitude of R.