Sums of Squares, Cubes, and Higher Powers

William Jagy
Irving Kaplansky

Abstract

Any integer is expressible as a sum of two squares and a cube, mixed signs being allowed. We study the analogous question for a square and two cubes, and obtain an affirmative answer in the range from $-4{,}000{,}000$ to $2{,}000{,}000$. For two squares and a cube with everything positive, computations support the possibility that there are only finitely many exceptions. However, $x^2 + y^2 + z^9$ admits infinitely many positive exceptions.