Circle Packing: Experiments in Discrete Analytic Function Theory

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Abstract

Circle packings are configurations of circles with specified patterns of tangency, and lend themselves naturally to computer experimentation and visualization. Maps between them display, with surprising faithfulness, many of the geometric properties associated with classical analytic functions. This paper introduces the fundamentals of an emerging "discrete analytic function theory" and investigates connections with the classical theory. It also describes experiments, ranging from a setup to gather evidence in support of a conjectured discrete Koebe $\frac{1}{4}$ theorem to a multigrid method for computing discrete approximations of classical analytic functions. These experiments were performed using CirclePack, a software package described in the paper and available free of charge.