Catalan's Equation Has No New Solution with Either Exponent Less Than 10651

Maurice Mignotte
Yves Roy

Abstract

We consider Catalan's equation $x^p - y^q = 1$ (where all variables are integers and p, q are greater than 1), which has the obvious solution $9 - 8 = 1$. Are there others? Applying old and new theoretical results to a systematic computation, we were able to improve on recent work of Mignotte and show that Catalan's equation has only the obvious solutions when $\min\{p, q\} < 10651$. Two crucial tools used are a new result of Laurent, Mignotte, and Nesterenko on linear forms of logarithms, and a criterion obtained by W. Schwarz in 1994.