On a Conjecture of Carathéodory: Analyticity Versus Smoothness

Federico Sánchez-Bringas

Abstract

We show that, under mild nonflatness conditions, for any $r \geq 3$ and any C^r-immersion of a surface into $\mathbb{R}^3$ with an isolated umbilic point there exist an analytic surface with an isolated umbilic of the same index. The connection of this with Carathéodory's Conjecture on umbilics is discussed.