Packing Lines, Planes, etc.: Packings in Grassmannian Spaces

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Abstract

We address the question: How should N n-dimensional subspaces of m-dimensional Euclidean space be arranged so that they are as far apart as possible? The results of extensive computations for modest values of N, n, m are described, as well as a reformulation of the problem that was suggested by these computations. The reformulation gives a way to describe n-dimensional subspaces of m-space as points on a sphere in dimension $\frac{1}{2}(m-1)(m+2)$, which provides a (usually) lower-dimensional representation than the Plücker embedding, and leads to a proof that many of the new packings are optimal. The results have applications to the graphical display of multi-dimensional data via Asimov's grand tour method.