Growth Functions and Automatic Groups

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Abstract

In this paper we study growth functions of automatic and hyperbolic groups. In addition to standard growth functions, we also want to count the number of finite graphs isomorphic to a given finite graph in the ball of radius n around the identity element in the Cayley graph. This topic was introduced to us by K. Saito. We report on fast methods to compute the growth function once we know the automatic structure. We prove that for a geodesic automatic structure, the growth function for any fixed finite connected graph is a rational function. For a word-hyperbolic group, we show that one can choose the denominator of the rational function independently of the finite graph.