EMPIRICALLY DETERMINED APÉRY-LIKE FORMULAE FOR $\zeta(4n+3)$

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Abstract. Some rapidly convergent formulae for special values of the Riemann zeta function are given. We obtain a generating function formula for $\zeta(4n+3)$ that generalizes Apéry's series for $\zeta(3)$, and appears to give the best possible series relations of this type, at least for $n < 12$. The formula reduces to a finite but apparently nontrivial combinatorial identity. The identity is equivalent to an interesting new integral evaluation for the central binomial coefficient. We outline a new technique for transforming and summing certain infinite series. We also derive a formula that provides strange evaluations of a large new class of nonterminating hypergeometric series.

[Editor's Note: The beautiful formulas in this paper are no longer conjectural.]