COMPUTER SEARCH FOR NILPOTENT COMPLEXES

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Abstract. The concept of nilpotency for a topological space is a generalization of simple connectivity. That it is a fruitful generalization was shown by Dror, Kan, Bousfield, Hilton, and others. In 1977 Brown and Kahn proved that the dimension of a nilpotent complex can be read from the ordinary homology groups, just as in the case of a simply connected complex. They also showed that if a nilpotent complex has finite and nontrivial fundamental group, its dimension must be at least 3.

In 1985 Lewis showed that for any finite nilpotent group there is a (not necessarily finite) three-dimensional nilpotent complex with that fundamental group. The smallest finite nilpotent group for which it was unknown whether a *finite* three-dimensional nilpotent complex exists was $\mathbb{Z}_2 \oplus \mathbb{Z}_6$.

The authors, together with a team of undergraduate students at Fordham University, used computers to search for three-dimensional finite nilpotent complexes over groups of the form $\mathbb{Z}_n \oplus \mathbb{Z}_m$. Such complexes were eventually found for $\mathbb{Z}_2 \oplus \mathbb{Z}_6$, $\mathbb{Z}_2 \oplus \mathbb{Z}_{10}$, and $\mathbb{Z}_3 \oplus \mathbb{Z}_6$.

This article describes the strategy for constructing nilpotent complexes of dimension three, and some of the issues in implementing the computer search. The main computational issues are "normalizing" matrices, especially to the Smith normal form, and mapping matrices over $\mathbb{Z}$ to matrices over $\mathbb{Z}_p$ for various primes p. We conclude with a summary of the complexes discovered and open questions.