INVARIANTS OF THREE-MANIFOLDS, UNITARY REPRESENTATIONS OF THE MAPPING CLASS GROUP, AND NUMERICAL CALCULATIONS

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Abstract. Based on previous results of the two first authors, it is shown that the combinatorial construction of invariants of compact, closed three-manifolds by Turaev and Viro as state sums in terms of quantum $6j$-symbols for $\mathrm{SL}_q(2,\mathbb{C})$ at roots of unity leads to the unitary representation of the mapping class group found by Moore and Seiberg. Via a Heegaard decomposition this invariant may therefore be written as the absolute square of a certain matrix element of a suitable group element in this representation. For an arbitrary Dehn surgery on a figure-eight knot we provide an explicit form for this matrix element involving just one $6j$-symbol. This expression is analyzed numerically and compared with the conjectured large $k = r - 2$ asymptotics of the Chern–Simons–Witten state sum, whose absolute square is the Turaev–Viro state sum. In particular we find numerical agreement concerning the values of the Chern–Simons invariants for the flat SU(2)-connections as predicted by the asymptotic expansion of the state sum with analytical results found by Kirk and Klassen.