# HYPERELLIPTIC SIMPLE FACTORS OF $J_0(N)$ WITH DIMENSION AT LEAST 3

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ABSTRACT. We develop algorithms for three problems. Starting with a complex torus of dimension $g \geq 2$, isomorphic to a principally polarized, simple abelian variety $A/\mathbb{C}$, the first problem is to find an algorithmic solution of the hyperelliptic Schottky problem: Is there a hyperelliptic curve $C$ of genus $g$ whose jacobian variety $\mathfrak{J}_C$ is isomorphic to $A$ over $\mathbb{C}$? Our solution is based on an article by Poor. If such a hyperelliptic curve $C$ exists, the next problem is the construction of the Rosenhain model $C : Y^2 = X(X-1)(X-\lambda_1)(X-\lambda_2) \ \ldots \ (X-\lambda_{2g-1})$ for pairwise distinct numbers $\lambda_j \in \mathbb{C} \setminus \{0,\, 1\}$. Applying the theory of hyperelliptic theta functions we show that these numbers $\lambda_j$ can easily be computed by using theta constants with even characteristics. If the abelian variety $A$ is defined over a field $k$ (this field could be the field of rational numbers, an algebraic number field of low degree, or a finite field), we show only in the case $k = \mathbb{Q}$ for simplicity, how a method of Mestre can be generalized to get a minimal equation over $\mathbb{Z}\left[\frac{1}{2}\right]$ for the hyperelliptic curve $C$ with jacobian variety $\mathfrak{J}_C \cong_{\mathbb{C}} A$. This is our third problem. For some hyperelliptic, principally polarized and simple factors with dimension $g = 3,\, 4,\, 5$ of the jacobian variety $J_0(N) = \mathfrak{J}_{X_0(N)}$ of the modular curve $X_0(N)$ we compute the corresponding curve equations by applying our algorithms to this special situation.