MAHLER'S MEASURE AND SPECIAL VALUES OF L-FUNCTIONS

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ABSTRACT. If $P(x_1, \ldots, x_n)$ is a polynomial with integer coefficients, the Mahler measure $M(P)$ of P is defined to be the geometric mean of $|P|$ over the n-torus $\mathbb{T}^n$. For $n = 1$, $M(P)$ is an algebraic integer, but for $n > 1$, there is reason to believe that $M(P)$ is usually transcendental. For example, Smyth showed that $\log M(1 + x + y) = L'(\chi_{-3}, -1)$, where χ_{-3} is the odd Dirichlet character of conductor 3. Here we will describe some examples for which it appears that $\log M(P(x, y)) = r\,L'(E, 0)$, where E is an elliptic curve and r is a rational number, often either an integer or the reciprocal of an integer. Most of the formulas we discover have been verified numerically to high accuracy but not rigorously proved.