EMBEDDING THE $3x+1$ CONJECTURE IN A $3x+d$ CONTEXT

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Abstract. Recall the well-known $3x+1$ conjecture: if $T(n) = (3n+1)/2$ for n odd and $T(n) = n/2$ for n even, repeated application of T to any positive integer eventually leads to the cycle

$$\{1 \to 2 \to 1\}.$$

We study a natural generalization of the function T, where instead of $3n+1$ one takes $3n+d$, for d equal to -1 or to an odd positive integer not divisible by 3. With this generalization new cyclic phenomena appear, side by side with the general convergent dynamics typical of the $3x+1$ case. Nonetheless, experiments suggest the following conjecture: For any odd $d \geq -1$ not divisible by 3 there exists a finite set of positive integers such that iteration of the $3x+d$ function eventually lands in this set.

Along with a new boundedness result, we present here an improved formalism, more clear-cut and better suited for future experimental research.