CONSTANT MEAN CURVATURE SURFACES WITH TWO ENDS IN HYPERBOLIC SPACE

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Abstract. We investigate the close relationship between minimal surfaces in Euclidean three-space and surfaces of constant mean curvature 1 in hyperbolic three-space. Just as in the case of minimal surfaces in Euclidean three-space, the only complete connected embedded surfaces of constant mean curvature 1 with two ends in hyperbolic space are well-understood surfaces of revolution: the catenoid cousins.

In contrast to this, we show that, unlike the case of minimal surfaces in Euclidean three-space, there do exist complete connected immersed surfaces of constant mean curvature 1 with two ends in hyperbolic space that are not surfaces of revolution: the genus-one catenoid cousins. These surfaces are of interest because they show that, although minimal surfaces in Euclidean three-space and surfaces of constant mean curvature 1 in hyperbolic three-space are intimately related, there are essential differences between these two sets of surfaces. The proof we give of existence of the genus-one catenoid cousins is a mathematically rigorous verification that the results of a computer experiment are sufficiently accurate to imply existence.