PRIME PERCOLATION

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ABSTRACT. This paper examines the question of whether there is an unbounded walk of bounded step size along Gaussian primes. Percolation theory predicts that for a low enough density of random Gaussian integers no walk exists, which suggests that no such walk exists along prime numbers, since they have arbitrarily small density over large enough regions. In analogy with the Cramér conjecture, I construct a random model of Gaussian primes and show that an unbounded walk of step size $k\sqrt{\log|z|}$ at z exists with probability 1 if $k > \sqrt{2\pi\lambda_c}$, and does not exist with probability 1 if $k < \sqrt{2\pi\lambda_c}$, where $\lambda_c \approx 0.35$ is a constant in continuum percolation, and so conjecture that the critical step size for Gaussian primes is also $\sqrt{2\pi\lambda_c \log|z|}$.