GALOIS REPRESENTATIONS, HECKE OPERATORS, AND THE MOD-p COHOMOLOGY OF GL$(3, \mathbb{Z})$ WITH TWISTED COEFFICIENTS

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Abstract. We compute the degree 3 homology of $GL(3, \mathbb{Z})$ with coefficients in the module of homogeneous polynomials in three variables of degree g over $\mathbb{F}_p$, for $g \leq 200$ and $p \leq 541$. The homology has a “boundary part” and a “quasicuspidal” part which we determine.

By conjecture a Hecke eigenclass in the homology has an attached Galois representation into $GL(3, \bar{\mathbb{F}}_p)$. The conjecture is proved for the boundary part and explored experimentally for the quasicuspidal part.