SINGULAR SEPARATRIX SPLITTING AND THE MELNIKOV METHOD: AN EXPERIMENTAL STUDY

AMADEU DELSHAMS AND RAFAEL RAMÍREZ-ROS

ABSTRACT. We consider families of analytic area-preserving maps depending on two parameters: the perturbation strength ε and the characteristic exponent h of the origin. For $\varepsilon = 0$, these maps are integrable with a separatrix to the origin, whereas they asymptote to flows with homoclinic connections as $h \to 0^+$. For fixed $\varepsilon \neq 0$ and small h, we show that these connections break up. The area of the lobes of the resultant turnstile is given asymptotically by $\varepsilon \exp(-\pi^2/h)\Theta^\varepsilon(h)$, where $\Theta^\varepsilon(h)$ is an even Gevrey-1 function such that $\Theta^\varepsilon(0) \neq 0$ and the radius of convergence of its Borel transform is $2\pi^2$. As $\varepsilon \to 0$, the function Θ^ε tends to an entire function Θ^0. This function Θ^0 agrees with the one provided by Melnikov theory, which cannot be applied directly, due to the exponentially small size of the lobe area with respect to h.

These results are supported by detailed numerical computations; we use multiple-precision arithmetic and expand the local invariant curves up to very high order.