ON THE VOLUME OF THE POLYTOPE OF DOUBLY STOCHASTIC MATRICES

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ABSTRACT. We study the calculation of the volume of the polytope B_n of $n \times n$ doubly stochastic matrices (real nonnegative matrices with row and column sums equal to one). We describe two methods. The first involves a decomposition of the polytope into simplices. The second involves the enumeration of "magic squares", that is, $n \times n$ nonnegative integer matrices whose rows and columns all sum to the same integer.

We have used the first method to confirm the previously known values through $n = 7$. This method can also be used to compute the volumes of faces of B_n. For example, we have observed that the volume of a particular face of B_n appears to be a product of Catalan numbers. We have used the second method to find the volume for $n = 8$, which we believe was not previously known.