THE $3n+1$-PROBLEM AND HOLOMORPHIC DYNAMICS

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ABSTRACT. The $3n+1$-problem is the following iterative procedure on the positive integers: the integer n maps to $n/2$ or $3n+1$, depending on whether n is even or odd. It is conjectured that every positive integer will be eventually periodic, and the cycle it falls onto is $1 \mapsto 4 \mapsto 2 \mapsto 1$. We construct entire holomorphic functions that realize the same dynamics on the integers and for which all the integers are in the Fatou set. We show that no integer is in a Baker domain (domain at infinity). We conclude that any integer that is not eventually periodic must be in a wandering domain.