ON CYCLOTOMIC POLYNOMIALS WITH ± 1 COEFFICIENTS

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Abstract. We characterize all cyclotomic polynomials of even degree with coefficients restricted to the set $\{+1, -1\}$. In this context a cyclotomic polynomial is any monic polynomial with integer coefficients and all roots of modulus 1. *Inter alia* we characterize all cyclotomic polynomials with odd coefficients.

The characterization is as follows. A polynomial $P(x)$ with coefficients ± 1 of even degree $N-1$ is cyclotomic if and only if

$$P(x) = \pm \Phi_{p_1}(\pm x)\, \Phi_{p_2}(\pm x^{p_1}) \cdots \Phi_{p_r}(\pm x^{p_1 p_2 \cdots p_{r-1}}),$$

where $N = p_1 p_2 \cdots p_r$ and the p_i are primes, not necessarily distinct, and where $\Phi_p(x) := (x^p - 1)/(x-1)$ is the p-th cyclotomic polynomial.

We conjecture that this characterization also holds for polynomials of odd degree with ± 1 coefficients. This conjecture is based on substantial computation plus a number of special cases.

Central to this paper is a careful analysis of the effect of Graeffe's root squaring algorithm on cyclotomic polynomials.