THE BLASIUS FUNCTION IN THE COMPLEX PLANE

JOHN P. BOYD

ABSTRACT. The Blasius function, denoted by f, is the solution to a simple nonlinear boundary layer problem, a third order ordinary differential equation on $x \in [0, \infty]$. In this work, we calculate several numerical constants, such as the second derivative of f at the origin and the two parameters of the linear asymptotic approximation to f, to at least eleven digits. Although the Blasius function is unbounded, we nevertheless derive an expansion in rational Chebyshev functions TL_j which converges exponentially fast with the truncation, and tabulate enough coefficients to compute f and its derivatives to about nine decimal places for all positive real x. The power series of f has a finite radius of convergence, but the Euler-accelerated expansion is apparently convergent for all real x. We show that the singularities, which are first order poles to lowest order, have an infinite series of cosine-of-a-logarithm corrections. Lastly, we chart the behavior of f in the complex plane and conjecture that all singularities lie within three narrow sectors.