# ON THE VOLUME OF A CERTAIN POLYTOPE

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ABSTRACT. Let $n \geq 2$ be an integer and consider the set $T_n$ of $n \times n$ permutation matrices $\pi$ for which $\pi_{ij} = 0$ for $j \geq i + 2$.

We study the convex hull $P_n$ of $T_n$, a polytope of dimension $\binom{n}{2}$. We provide evidence for several conjectures involving $P_n$, including Conjecture 1: Let $v_n$ denote the minimum volume of a simplex with vertices in the affine lattice spanned by $T_n$. Then the volume of $P_n$ is $v_n$ times the product

$$\prod_{i=0}^{n-2} \frac{1}{i+1}\binom{2i}{i}$$

of the first $n-1$ Catalan numbers.

We also give a related result on the Ehrhart polynomial of $P_n$.