PERIODIC AUTOMORPHISMS OF SURFACES: INVARIANT CIRCLES AND MAXIMAL ORDERS

HANSJÖRG GEIGES AND DIEGO RATTAGGI

Abstract. W. H. Meeks has asked the following question: For what g does every (orientation preserving) periodic automorphism of a closed orientable surface of genus g have an invariant circle? A variant of this question due to R. D. Edwards asks for the existence of invariant *essential* circles. Using a construction of Meeks we show that the answer to his question is negative for all but 43 values of $g \leq 10000$, all of which lie below $g = 105$. We then show that the work of S. C. Wang on Edwards' question generalizes to nonorientable surfaces and automorphisms of *odd* order. Motivated by this, we ask for the maximal *odd* order of a periodic automorphism of a given nonorientable surface. We obtain a fairly complete answer to this question and also observe an amusing relation between this order and Fermat primes.