A CHAOTIC COUSIN OF CONWAY'S RECURSIVE SEQUENCE

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ABSTRACT. I introduce the recurrence

$$D(n) = D(D(n-1)) + D(n-1 - D(n-2)),$$

$D(1) = D(2) = 1$, and study it by means of computer experiments. The definition of $D(n)$ has some similarity to that of Conway's sequence defined by $a(n) = a(a(n-1)) + a(n - a(n-1))$, $a(1) = a(2) = 1$. However, unlike the completely regular and predictable behaviour of $a(n)$, the D-numbers exhibit chaotic patterns. In its statistical properties, the D-sequence shows striking similarities with Hofstadter's $Q(n)$-sequence, given by $Q(n) = Q(n - Q(n-1)) + Q(n - Q(n-2))$, $Q(1) = Q(2) = 1$. Compared to the Hofstadter sequence, D shows higher structural order. It is organized in well-defined "generations", separated by smooth and predictable regions. The article is complemented by a study of two further recurrence relations with definitions similar to those of the Q-numbers. There is some evidence that the different sequences studied share a universality class.