FREQUENCY LOCKING ON THE BOUNDARY OF THE BARYCENTRE SET

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ABSTRACT. We consider the doubling map $T : z \mapsto z^2$ of the circle. For each T-invariant probability measure μ we define its barycentre $b(\mu) = \int_{S^1} z \, d\mu(z)$, which describes its average weight around the circle. We study the set Ω of all such barycentres, a compact convex set with nonempty interior. Its boundary has a countable dense set of points of nondifferentiability, the worst possible regularity for the boundary of a convex set. We explain this behaviour in terms of the frequency locking of rotation numbers for a certain class of invariant measures, each supported on the closure of a Sturmian orbit.