REAL SCHUBERT CALCULUS: POLYNOMIAL SYSTEMS AND A CONJECTURE OF SHAPIRO AND SHAPIRO

FRANK SOTTILE

ABSTRACT. Boris and Michael Shapiro have a conjecture concerning the Schubert calculus and real enumerative geometry and which would give infinitely many families of zero-dimensional systems of real polynomials (including families of overdetermined systems)—all of whose solutions are real. It has connections to the pole placement problem in linear systems theory and to totally positive matrices. We give compelling computational evidence for its validity, prove it for infinitely many families of enumerative problems, show how a simple version implies more general versions, and present a counterexample to a general version of their conjecture.