POLYNOMIALS WITH HEIGHT 1 AND PRESCRIBED VANISHING AT 1

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ABSTRACT. We study the minimal degree $d(m)$ of a polynomial with all coefficients in $\{-1, 0, 1\}$ and a zero of order m at 1. We determine $d(m)$ for $m \leq 10$ and compute all the extremal polynomials. We also determine the minimal degree for $m = 11$ and $m = 12$ among certain symmetric polynomials, and we find explicit examples with small degree for $m \leq 21$. Each of the extremal examples is a pure product polynomial. The method uses algebraic number theory and combinatorial computations and relies on showing that a polynomial with bounded degree, restricted coefficients, and a zero of high order at 1 automatically vanishes at several roots of unity.