INFINITE REGULAR HEXAGON SEQUENCES ON A TRIANGLE

ALVY RAY SMITH

ABSTRACT. The well-known dual pair of Napoleon equilateral triangles intrinsic to each triangle is extended to infinite sequences of them, shown to be special cases of infinite regular hexagon sequences on each triangle. A set of hexagon-to-hexagon transformations, the hex operators, is defined for this purpose, a set forming an abelian monoid under function composition. The sequences result from arbitrary strings of hex operators applied to a particular truncation of a given triangle to a hexagon. The deep structure of the sequence constructions reveals surprising infinite sequences of nonconcentric, symmetric equilateral triangle pairs parallel to one of the sequences of hexagons and provides the most visually striking contribution. Extensive experimentation with a plane geometry educational program inspired all theorems, proofs of which utilize eigenvector analysis of polygons in the complex plane.