Weaire–Phelan	0.764
β-uranium	0.762
Kelvin	0.757
C15	0.749
f.c.c. (face-centered cubic)	0.7405

TABLE 1. The isoperimetric quotient $36\pi V^2/A^3$ (figure of merit for surface area minimisation) of some rival dry foam structures. The f.c.c. structure is unstable in this limit.

3. WET FOAM STRUCTURES

The (volume) liquid fraction of a foam may be designated by φ_{liq}. The calculations of the previous section are for dry foam, that is, φ_{liq}=0. For any given structure there is a maximum value of φ_{liq}, defining the wet foam limit, beyond which the foam bubbles become separated. This has been called the rigidity loss transition by Bolton and Weaire [1990], and hitherto explored in detail only for two dimensions [Weaire et al. 1994]. For b.c.c. and f.c.c., the critical liquid fractions are 0.32 and 0.26, while for the WP structure it is 0.47. Consideration of the range of φ_{liq} intermediate between the two limits raises a number of questions [Weaire 1994]. What is the ideal ordered structure for each value of φ_{liq}? How are the transitions between rival structures dictated by instabilities and topological changes?

In order to address such questions, we adapted the Evolver to allow both for thin film surfaces where bubbles are in contact and for the rest of the bubble surfaces. The surface energy to be associated with the double-sided films is clearly twice that of the bubble surfaces. Figures 7–5 present typical examples of equilibrium structures generated in this way. In each case the individual cells have rounded edges, in contrast to those of Figures 1, 3 and 4. A complementary view of the structure is that based on the liquid-filled regions (the Plateau borders), as shown in Figure 8.

We can extend the definition of the isoperimetric quotient given above to the case of wet foams by taking V to be the initial cell volume of the dry foam structure. Generalised isoperimetric quotients calculated using the program are presented in Figure 9. In obtaining these data, we fixed the

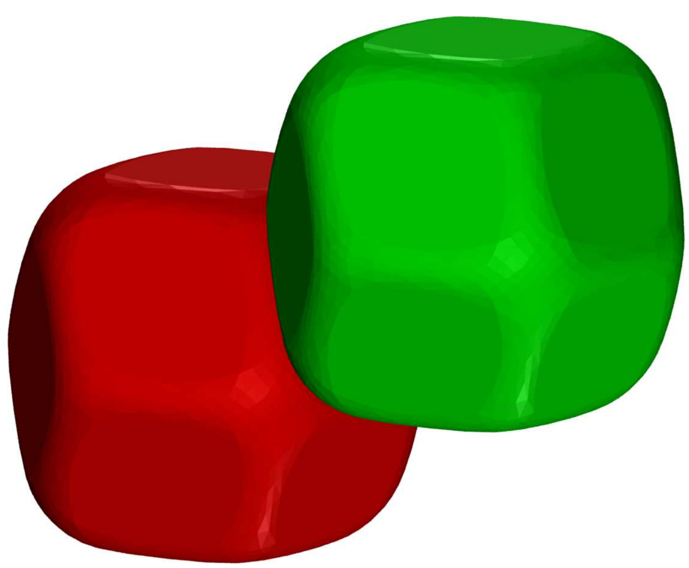

FIGURE 5. The f.c.c. structure for a liquid fraction of approximately 1%. Note the eightfold vertices, which are unstable in the limit $\varphi_{\mathrm{liq}} \to 0$. These and the following images were generated using Geomview [Phillips et al. 1993].