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BOUNDARY INTEGRAL METHODS FOR HARMONIC DIFFERENTIAL FORMS IN LIPSCHITZ DOMAINS

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ABSTRACT. A layer potential based approach for boundary value problems for harmonic differential forms in nonsmooth domains is developed. This allows a complete and unified treatment of several fundamental problems in potential theory.

§1. INTRODUCTION

In this note we report on recent progress in the study of boundary value problems for harmonic differential forms on Lipschitz domains by means of layer potential methods. The main issue which is addressed here is that of the effectiveness of the boundary integral methods in the higher degree context, arbitrary topology and in the presence of singularities.

Our treatment unifies and generalizes several major directions in potential theory. Most notably, our theory encompasses both the classical theory of harmonic integrals (or generalized potential theory) initiated by Hodge in the 1930's and studied at length by many authors thereafter (cf., e.g., the monographs [Ho], [Mo], [Con], [Ta] and the references therein) as well as the more recent theory of boundary value problems for harmonic scalar-valued functions in Lipschitz domains emerging from Calderón's program in the 1960's; see [Dah], [FaJoRi], [JeKe], [Ve].

The rather very general setting within which we formulate and solve these problems introduces new, significant difficulties which can be roughly categorized as having (a) analytical nature, and (b) topological nature. Obstacles in the first category arise as a result of allowing higher degree differential forms on domains with only Lipschitz continuous boundaries and square-integrable boundary data. Those in the second category occur in connection with the process of "patching" together local results in order to deal with global versions of our BVP's.

We develop a very effective, hands-on approach for these problems, and many of our results are new even in the smooth context. In fact, this allows us for the first time to undertake a systematic study of all natural boundary conditions for the Laplace operator acting on differential forms. However, due to obvious space limitations, here we explain only one example, playing a paradigm role for the entire theory.

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§2. Statement of a boundary value problem

We shall work with differential forms for which we employ fairly standard notation. In particular, d, δ, $*$ stand for the exterior derivative, co-derivative and Hodge-$*$ operator, respectively. Also, $\wedge$ and $\vee$ denote, respectively, the exterior and interior product of forms.

Let Ω be an arbitrary bounded Lipschitz domain in $\mathbb{R}^m$ (occasionally denoted by Ω_+), i.e. a domain whose boundary is locally given by graphs of Lipschitz functions. The outward unit normal n will be canonically identified with the 1-form $\sum_j n_j dx_j$. For each integer $0 \leq l \leq m$, we introduce the space of *tangential* square-integrable (with respect to the surface measure $d\sigma$) l-forms defined on $\partial\Omega$ by

$$L^2_{tan}(\partial\Omega, \Lambda^l \mathbb{R}^m) := \{A \in L^2(\partial\Omega, \Lambda^l \mathbb{R}^m),\ n \vee A = 0 \text{ a.e. on } \partial\Omega\}.$$

Here "a.e." is taken with respect to the canonical surface measure $d\sigma$. Let $\langle \cdot, \cdot \rangle$ stand for the usual (pointwise) Euclidean pairing of forms. An l-form A in the space $L^2_{tan}(\partial\Omega, \Lambda^l \mathbb{R}^m)$ is said to have its *boundary (exterior) co-derivative* in L^2 if there exists an $(l-1)$-form in $L^2(\partial\Omega, \Lambda^{l-1}\mathbb{R}^m)$, which we denote by $\delta_\partial A$, such that

$$\int_{\partial\Omega} \langle d\psi, A\rangle d\sigma = \int_{\partial\Omega} \langle \psi, \delta_\partial A\rangle d\sigma \quad \text{for any } \psi \in C^\infty(\mathbb{R}^m, \Lambda^{l-1}\mathbb{R}^m).$$

We set $L^{2,\delta}_{tan}(\partial\Omega, \Lambda^l \mathbb{R}^m) := \{A \in L^2_{tan}(\partial\Omega, \Lambda^l \mathbb{R}^m);\ \delta_\partial A \in L^2(\partial\Omega, \Lambda^{l-1}\mathbb{R}^m)\}$, and equip it with the natural norm $\|A\|_{L^{2,\delta}_{tan}} := \|A\|_{L^2(\partial\Omega)} + \|\delta_\partial A\|_{L^2(\partial\Omega)}$. We shall also make use of the (closed) subspace $L^{2,0}_{tan}(\partial\Omega, \Lambda^l \mathbb{R}^m)$ of $L^{2,\delta}_{tan}(\partial\Omega, \Lambda^l \mathbb{R}^m)$ given by

$$L^{2,0}_{tan}(\partial\Omega, \Lambda^l \mathbb{R}^m) := \left\{A \in L^{2,\delta}_{tan}(\partial\Omega, \Lambda^l \mathbb{R}^m);\ \delta_\partial A = 0\right\}.$$

Consider the boundary value problem

$$(BVP1_l) \begin{cases} F \in C^\infty(\Omega, \Lambda^l \mathbb{R}^m), \\ \triangle F = 0 \text{ in } \Omega, \\ \mathcal{N}(F), \mathcal{N}(dF) \in L^2(\partial\Omega), \\ n \vee F|_{\partial\Omega} = A \in L^2_{tan}(\partial\Omega; \Lambda^{l-1}\mathbb{R}^m), \\ n \vee (dF)|_{\partial\Omega} = B \in L^2_{tan}(\partial\Omega, \Lambda^l \mathbb{R}^m). \end{cases}$$

Here A and B are some a priori given tangential forms on the boundary and the restriction to the boundary is taken in the nontangential pointwise sense. Also, $\mathcal{N}(\cdot)$ is the usual nontangential maximal operator and $\triangle$ is the Laplacian in $\mathbb{R}^m$.

In order to state our first result, for $0 \leq l \leq m$ we introduce

$$\mathcal{H}^l_\vee(\Omega) := \{E \in C^\infty(\Omega, \Lambda^l \mathbb{R}^m);\ \mathcal{N}(E) \in L^2(\partial\Omega),$$
$$dE = 0 \text{ and } \delta E = 0 \text{ in } \Omega,\ n \vee E|_{\partial\Omega} = 0 \text{ on } \partial\Omega\},$$

and recall that S_l stands for the usual single layer potential operator for $\partial\Omega$ acting (componentwise) on l-forms on $\partial\Omega$.

Theorem 1. *With the above notation we have:*

1. *A solution of $(BVP1_l)$ exists if and only if B satisfies the compatibility condition $B \in \{E|_{\partial\Omega};\ E \in \mathcal{H}^l_\vee(\Omega)\}^\perp$.*
2. *The dimension of the space of null-solutions for the homogeneous problem is $b_l(\Omega)$, the l-th Betti number of Ω, and, in fact, this space coincides precisely with $\mathcal{H}^l_\vee(\Omega)$. In particular, the boundary data determine dF and δF uniquely.*

If the compatibility condition is satisfied, then every solution has the form $F = S_l U_1 + dS_{l-1}U_2 + \delta S_{l+1}U_3$ *with* $U_1 \in L^2(\partial\Omega, \Lambda^l\mathbb{R}^m)$, $U_2 \in L^2_{tan}(\partial\Omega, \Lambda^{l-1}\mathbb{R}^m)$, $U_3 \in *L^{2,0}_{tan}(\partial\Omega, \Lambda^{m-l-1}\mathbb{R}^m)$, *and*

$$\|\mathcal{N}(dF)\|_{L^2(\partial\Omega)} \leq C\|B\|_{L^2_{tan}(\partial\Omega,\Lambda^l\mathbb{R}^m)}.$$

Moreover, we have the following regularity statements:

3. $\mathcal{N}(\delta dF) \in L^2(\partial\Omega)$ *if and only if* $B \in L^{2,\delta}_{tan}(\partial\Omega, \Lambda^l\mathbb{R}^m)$. *In this case*
$$\|\mathcal{N}(\delta dF)\|_{L^2(\partial\Omega)} \leq C\|B\|_{L^{2,\delta}_{tan}(\partial\Omega,\Lambda^l\mathbb{R}^m)}.$$
4. $\delta dF = 0$ *in* Ω *if and only if* $B \in L^{2,0}_{tan}(\partial\Omega, \Lambda^l\mathbb{R}^m)$. *In particular, for* B *in* $L^{2,0}_{tan}(\partial\Omega, \Lambda^l\mathbb{R}^m)$ *the problem* $(BVP1_l)$ *becomes*
$$(BVP2_l)\begin{cases} F \in C^\infty(\Omega, \Lambda^l\mathbb{R}^m), \\ \triangle F = 0 \text{ in } \Omega, \\ \delta dF = 0 \text{ in } \Omega, \\ \mathcal{N}(F),\, \mathcal{N}(dF) \in L^2(\partial\Omega), \\ n \vee F|_{\partial\Omega} = A \in L^2_{tan}(\partial\Omega; \Lambda^{l-1}\mathbb{R}^m), \\ n \vee (dF)|_{\partial\Omega} = B \in L^{2,0}_{tan}(\partial\Omega, \Lambda^l\mathbb{R}^m). \end{cases}$$
5. $\mathcal{N}(\delta F) \in L^2(\partial\Omega)$ *if and only if* $A \in L^{2,\delta}_{tan}(\partial\Omega, \Lambda^{l-1}\mathbb{R}^m)$. *Moreover, for* $A \in L^{2,\delta}_{tan}(\partial\Omega, \Lambda^{l-1}\mathbb{R}^m)$, *there holds*
$$\|\mathcal{N}(\delta F)\|_{L^2(\partial\Omega)} \leq C\left(\|A\|_{L^{2,\delta}_{tan}(\partial\Omega,\Lambda^{l-1}\mathbb{R}^m)} + \|B\|_{L^2_{tan}(\partial\Omega,\Lambda^l\mathbb{R}^m)}\right).$$
6. $dF = 0$ *in* Ω *if and only if* $B = 0$. *Furthermore, when* $B = 0$, *we can prescribe periods and have genuine uniqueness. More precisely, for any* $\beta_j \in \mathbb{R}$, $j = 1, ..., b_l(\Omega)$, *there exists a unique solution of*
$$(BVP3_l)\begin{cases} F \in C^\infty(\Omega, \Lambda^l\mathbb{R}^m), \\ \triangle F = 0 \text{ in } \Omega, \\ dF = 0 \text{ in } \Omega, \\ \mathcal{N}(F) \in L^2(\partial\Omega), \\ n \vee F|_{\partial\Omega} = A \in L^2_{tan}(\partial\Omega; \Lambda^{l-1}\mathbb{R}^m), \\ \int_{\gamma_j} \iota^* F = \beta_j,\ j = 1, ..., b_l(\Omega), \end{cases}$$
where $[\gamma_j]_{j=1,\dots,b_l(\Omega)}$ *is a basis of* $H^l_{sing}(\Omega;\mathbb{R})$, *the* l*-th singular homology group of* Ω *over the reals, and* $\iota : \gamma_j \to \Omega$ *is the inclusion for all* j*'s.*
7. $\delta F = 0$ *if and only if* $A \in L^{2,0}_{tan}(\partial\Omega, \Lambda^{l-1}\mathbb{R}^m) \cap \{\mathcal{H}^{l-1}_{\vee}(\Omega)|_{\partial\Omega}\}^{\perp}$ *and* $B \in L^{2,0}_{tan}(\partial\Omega, \Lambda^l\mathbb{R}^m)$. *In fact, if* A *and* B *are as above, then* $(BVP1_l)$ *reduces to*
$$(BVP4_l)\begin{cases} F \in C^\infty(\Omega, \Lambda^l\mathbb{R}^m), \\ \triangle F = 0 \text{ in } \Omega, \\ \delta F = 0 \text{ in } \Omega, \\ \mathcal{N}(F),\, \mathcal{N}(dF) \in L^2(\partial\Omega), \\ n \vee F|_{\partial\Omega} = A \in L^{2,0}_{tan}(\partial\Omega, \Lambda^{l-1}\mathbb{R}^m), \\ n \vee dF|_{\partial\Omega} = B \in L^{2,0}_{tan}(\partial\Omega, \Lambda^l\mathbb{R}^m). \end{cases}$$

In particular, $B = 0$ forces $dF = 0$, and we can prescribe periods, in which case $(BVP4_l)$ becomes

$$(BVP5_l)\begin{cases} F \in C^\infty(\Omega, \Lambda^l\mathbb{R}^m), \\ \delta F = 0 \ in\ \Omega, \\ dF = 0 \ in\ \Omega, \\ \mathcal{N}(F) \in L^2(\partial\Omega), \\ n \vee F|_{\partial\Omega} = A \in L^{2,0}_{tan}(\partial\Omega, \Lambda^{l-1}\mathbb{R}^m), \\ \int_{\gamma_j} \iota^* F = \beta_j,\ j = 1, ..., b_l(\Omega). \end{cases}$$

Formulated as such, the problem $(BVP5_l)$ has a solution if and only if $A \in \{\mathcal{H}^{l-1}_\vee(\Omega)|_{\partial\Omega}\}^\perp$ and the solution is unique.

There is an analogous statement for the dual problem of $(BVP1_l)$, corresponding to an application of the Hodge star isomorphism. Also, similar results are valid for the exterior domain $\Omega_- := \mathbb{R}^m \setminus \bar{\Omega}$ (with appropriate decay conditions included).

It is both rewarding and illuminating to point out that $(BVP1_l)$ becomes the Dirichlet problem for the Laplacian (in slight disguise) for $l = m$ and, further, its so-called regular version if, in addition, $A \in L^{2,\delta}_{tan}(\partial\Omega, \Lambda^m\mathbb{R}^m)(= W^{1,2}(\partial\Omega)d\mathrm{Vol}_m)$. Also, $(BVP1_l)$ reduces precisely to the classical Neumann problem for the Laplacian in the case when $l = 0$. For Lipschitz domains, these problems have been first addressed in the work of B. Dahlberg, E. Fabes, D. Jerison, C. Kenig, G. Verchota, among others. The family $(BVP_l)_{0\le l\le m}$ also encompasses problems arising in static electromagnetism for Lipschitz domains in $\mathbb{R}^3$. This identification takes place at the level $l = 1$, when one canonically identifies differential forms of degree one with vector fields. See [MiMiPi].

For forms of arbitrary degree, so far the only reasonably well understood case of the problem $(BVP1_l)$ is when the domain Ω has a smooth boundary and when the boundary data are smooth as well (see the excellent exposition in [Ta] in this connection). Of course, in the smooth case, there are a number of specific techniques which are not available in the irregular setting like, e.g., a symbolic calculus for pseudo-differential operators, as well as various regularity results near the boundary which readily open the door for the applicability of the De Rham cohomology machinery. As many major tools valid in the smooth case break down in the presence of singularities, the connection between potential theory and topology is considerably more difficult to explore in the present, more general context.

§3. Boundary integral operators

In solving the above problems, we rely on boundary integral methods. A particularly important role is played by the family of operators $\{\pm\frac{1}{2}I + M_l\}_{0\le l\le m}$, where I is the identity and M_l is a singular integral operator defined by

$$M_l A(P) := \lim_{\epsilon\to 0}\left(n(P) \vee \int_{\substack{Q\in\partial\Omega \\ |P-Q|\ge\epsilon}} (d\Gamma)(P-Q) \wedge A(Q)\, d\sigma(Q)\right), \quad P \in \partial\Omega.$$

Here $\Gamma(X)$ is the canonical fundamental solution for the Laplacian in $\mathbb{R}^m$ and $A \in L^2_{tan}(\partial\Omega, \Lambda^l\mathbb{R}^m)$. It is important to note that the family of singular integral operators $\{M_l\}_{0\le l\le m}$ encompasses both the classical double layer potential operator

K as well as its adjoint K^*. Indeed, M_l can be canonically identified with K and K^* for $l = m-1$ and $l = 0$, respectively.

Theorem 2. *Let Ω be a bounded Lipschitz domain in $\mathbb{R}^m$ and $0 \le l \le m$. Then the following statements are valid:*

1. *The operators $\pm\frac{1}{2} + M_l$ are Fredholm with index zero on each of the spaces $L^2_{tan}(\partial\Omega, \Lambda^l\mathbb{R}^m)$, $L^{2,\delta}_{tan}(\partial\Omega, \Lambda^l\mathbb{R}^m)$, $L^{2,0}_{tan}(\partial\Omega, \Lambda^l\mathbb{R}^m)$. Furthermore, their kernels on these spaces coincide and, in fact,*

$$Ker\left(\pm\tfrac{1}{2}I + M_l; L^2_{tan}(\partial\Omega, \Lambda^l\mathbb{R}^m)\right) = \left\{n \vee E|_{\partial\Omega_\pm}; E \in \mathcal{H}^{l+1}_\wedge(\Omega_\pm)\right\}.$$

In particular, $dim\, Ker\left(\frac{1}{2}I + M_l; L^2_{tan}(\partial\Omega, \Lambda^l\mathbb{R}^m)\right) = b_{m-l-1}(\Omega)$, and

$$dim\, Ker\left(-\tfrac{1}{2}I + M_l; L^2_{tan}(\partial\Omega, \Lambda^l\mathbb{R}^m)\right) = b_l(\Omega).$$

Also,

$$\begin{aligned} &Image\left(\pm\tfrac{1}{2}I + M_l; L^2_{tan}(\partial\Omega, \Lambda^l\mathbb{R}^m)\right) \\ &\quad = \left\{A \in L^2_{tan}(\partial\Omega, \Lambda^l\mathbb{R}^m);\ A \perp E|_{\partial\Omega_\mp} \text{ for any } E \in \mathcal{H}^l_\vee(\Omega_\mp)\right\}. \end{aligned}$$

Similar descriptions are valid for the images of $\pm\frac{1}{2}I + M_l$ when acting on $L^{2,\delta}_{tan}(\partial\Omega, \Lambda^l\mathbb{R}^m)$ and $L^{2,0}_{tan}(\partial\Omega, \Lambda^l\mathbb{R}^m)$.

Also, the following operators are isomorphisms on the indicated spaces:

2. *$\pm\frac{1}{2}I + M_l$ acting on $\delta_\partial\left[L^{2,\delta}_{tan}(\partial\Omega, \Lambda^l\mathbb{R}^m)\right]$;*
3. *$\pm\frac{1}{2}I + M_l$ acting on*

$$\frac{L^{2,\delta}_{tan}(\partial\Omega, \Lambda^l\mathbb{R}^m)}{L^{2,0}_{tan}(\partial\Omega, \Lambda^l\mathbb{R}^m)} \quad and \quad \frac{L^{2}_{tan}(\partial\Omega, \Lambda^l\mathbb{R}^m)}{L^{2,0}_{tan}(\partial\Omega, \Lambda^l\mathbb{R}^m)};$$

4. *$\pm\frac{1}{2}I + M_l$ acting on*

$$\frac{L^{2}_{tan}(\partial\Omega, \Lambda^l\mathbb{R}^m)}{Ker(\pm\frac{1}{2}I + M_l)}, \quad \frac{L^{2,\delta}_{tan}(\partial\Omega, \Lambda^l\mathbb{R}^m)}{Ker(\pm\frac{1}{2}I + M_l)}, \quad and \quad \frac{L^{2,0}_{tan}(\partial\Omega, \Lambda^l\mathbb{R}^m)}{Ker(\pm\frac{1}{2}I + M_l)}.$$

5. *$\pm\frac{1}{2}I + M_l$ acting on $(n \vee *L^{2,0}_{tan}(\partial\Omega, \Lambda^{m-l}\mathbb{R}^m))^\perp$;*
6. *$n \vee S_l + \left(\pm\frac{1}{2}I + M_{l-1}\right)^{-1}[\delta_\partial(n \vee S_{l+1})(n \wedge S_l)]$ from $*Ker(\mp\frac{1}{2}I + M_{m-l})$ onto $Ker(\pm\frac{1}{2}I + M_{l-1})$, where the inverse operators $(\pm\frac{1}{2}I + M_{l-1})^{-1}$ are considered on the space $\delta_\partial\left[L^{2,\delta}_{tan}(\partial\Omega, \Lambda^{l-1}\mathbb{R}^m)\right]$ (cf. (2) above).*

As this theorem shows, there are natural obstructions to inverting the boundary layer potential operators $\pm\frac{1}{2}I + M_l$ on, e.g., $L^2_{tan}(\partial\Omega, \Lambda^l\mathbb{R}^m)$ which are expressed in the form of the nonvanishing of certain singular homology groups of the underlying domain. In particular, a too direct utilization of these operators in conjunction with $(BVP1_l)$ appears to require suitable topological restrictions. In this light, it is rather remarkable that such "side-effects" can be avoided making appropriate corrections (i.e., by adding further source terms).

§4. A regularity theorem

A basic result which allows us to relate boundary integral operators to the mechanism of (interior) Hodge type decompositions for forms with coefficients in $L^2(\Omega)$, is a certain regularity theorem which is interesting in its own rights and which we now describe.

Theorem 3. *For a differential form $E \in L^2(\Omega, \Lambda^l \mathbb{R}^m)$ with $dE \in L^2(\Omega, \Lambda^{l+1}\mathbb{R}^m)$, $\delta E \in L^2(\Omega, \Lambda^{l-1}\mathbb{R}^m)$ (d and δ are considered in some weak distributional sense) the following are equivalent:*

1. *$n \wedge E$, initially considered in the sense of distributions in the Sobolev space $W^{-\frac{1}{2},2}(\partial\Omega, \Lambda^{l+1}\mathbb{R}^m)$, actually belongs to $L^2(\partial\Omega, \Lambda^{l+1}\mathbb{R}^m)$;*
2. *$n \vee E$, initially considered in the sense of distributions in the Sobolev space $W^{-\frac{1}{2},2}(\partial\Omega, \Lambda^{l-1}\mathbb{R}^m)$, actually belongs to $L^2(\partial\Omega, \Lambda^{l-1}\mathbb{R}^m)$;*
3. *$E \in W^{\frac{1}{2},2}(\Omega, \Lambda^l \mathbb{R}^m)$.*

There are also natural accompanying estimates in each case. In particular,

$$\begin{aligned}\|E\|_{W^{\frac{1}{2},2}(\Omega,\Lambda^l\mathbb{R}^m)} \leq & C\left(\|E\|_{L^2(\Omega,\Lambda^l\mathbb{R}^m)} + \|dE\|_{L^2(\Omega,\Lambda^{l+1}\mathbb{R}^m)} + \|\delta E\|_{L^2(\Omega,\Lambda^{l-1}\mathbb{R}^m)}\right) \\ & + C\min\left\{\|n\wedge E\|_{L^2(\partial\Omega,\Lambda^{l+1}\mathbb{R}^m)}, \|n\vee E\|_{L^2(\partial\Omega,\Lambda^{l-1}\mathbb{R}^m)}\right\}.\end{aligned}$$

The exponent $\frac{1}{2}$ is sharp in the class of Lipschitz domains, and this is strongly contrasting the smooth case where $\frac{1}{2}$ may be replaced by 1.

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