On the Special Trace Decomposition Problem on Quaternion Structure

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Abstract

The trace decomposition of tensors was studied by D. Krupka and J. Mikeš. Krupka's theorem has been generalized to the case of spaces with complex structure in an article of J. Mikeš. These results are generalized to the case of general decomposition on quaternionic structure in this paper.

1 Introduction

Let E be a real n-dimensional vector space, E^p_q the vector space of tensors of type (p,q) on E. We choose a fixed basis of E. This basis determines a unique basis of E^p_q. The components with respect to this basis of any tensor T will be denoted $T^{i_1 i_2 \dots\, i_p}_{j_1 j_2 \dots\, j_q}$.

A tensor $T \in E^p_q$ is *traceless* if the following relationship is true

$$\forall k = \overline{1,p}; \quad \forall r = \overline{1,q}: \qquad T^{\dots i_{k-1} \alpha i_{k+1} \dots}_{\dots j_{r-1} \alpha j_{r+1} \dots} \;=\; 0.$$

The following theorem is Krupka's trace decomposition theorem. This theorem solves the problem of the existence and uniqueness of a decomposition of a tensor in which one term is traceless and the other terms are linear combinations of the Kronecker's δ-tensor. This theorem is proved in [4], [5].

Theorem 1 *Let $T^{i_1 i_2 \dots i_p}_{j_1 j_2 \dots j_q}$ be a tensor of type (p,q). If $n+1 \geq p+q$, then there exists a unique decomposition of T of the form*

$$T^{i_1 i_2 \dots i_p}_{j_1 j_2 \dots j_q} \;=\; B^{i_1 i_2 \dots i_p}_{j_1 j_2 \dots j_q} + \sum_{t=1}^{min\{p,q\}} \sum_{\bigoplus} \delta^{i_{\rho_1} i_{\rho_2} \dots i_{\rho_t}}_{j_{\sigma_1} j_{\sigma_2} \dots j_{\sigma_t}} B^{\{\rho_1 \rho_2 \dots \rho_t\} \dots}_{\{\sigma_1 \sigma_2 \dots \sigma_t\} \dots}\ ,$$

where

$$\bigoplus = \begin{cases} \forall \rho_1, \rho_2, \dots, \rho_t \in \overline{1,p} & (\rho_1 < \rho_2 < \dots < \rho_t)\ , \\ \forall \sigma_1, \sigma_2, \dots, \sigma_t \in \overline{1,q} & (\sigma_i \text{ are mutually different})\ , \end{cases}$$

and

$$\delta^{i_1 i_2 \dots i_t}_{j_1 j_2 \dots j_t} = \delta^{i_1}_{j_1} \delta^{i_2}_{j_2} \dots \delta^{i_t}_{j_t}, \text{ such that the tensors } B^{\dots}_{\dots}\, ; B^{\{\rho_1 \rho_2 \dots \rho_t\}}_{\{\sigma_1 \sigma_2 \dots \sigma_t\}} \text{ are traceless.}$$

Let F^i_j be an arbitrary affinor ($F^i_j \in E^1_1$ such as $F^\alpha_\alpha = 0$. A tensor $T \in E^p_q$ is called F-traceless if the following condition holds.

$$\forall k = \overline{1,p}; \quad \forall r = \overline{1,q}: \quad T^{\dots i_{k-1} \alpha i_{k+1} \dots}_{\dots j_{r-1} \beta j_{r+1} \dots} F^\beta_\alpha = 0, T^{\dots i_{k-1} \alpha i_{k+1} \dots}_{\dots j_{r-1} \alpha j_{r+1} \dots} = 0.$$

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Now we will study e-structures. For these structures the next condition is true

$$F^i_\alpha F^\alpha_j = e\delta^i_j\ ,$$

where $e = \pm 1$.

The following theorem is F-decomposition for e-structures. This theorem was proved by J. Mikeš in [5] and it generalizes Theorem 1.

Theorem 2 *Let T be a tensor of type (p,q). If $n > 2(p+q)$ then there exists a unique decomposition of T in the form*

$$T^{i_1 i_2 \dots i_p}_{j_1 j_2 \dots j_q} = B^{i_1 i_2 \dots i_p}_{j_1 j_2 \dots j_q} + \sum_{t=1}^{min\{p,q\}} \sum_{\bigoplus} \overset{\star}{Q}{}^{i_{\rho_1} i_{\rho_2} \dots i_{\rho_t}}_{j_{\sigma_1} j_{\sigma_2} \dots j_{\sigma_t}}\ \overset{\diamond}{B}{}^{\dots}_{\dots}\ , \tag{1}$$

where

$$\overset{\star}{Q}{}^{i_{\rho_1} i_{\rho_2} \dots\ i_{\rho_t}}_{j_{\sigma_1} j_{\sigma_2} \dots\ j_{\sigma_t}} \equiv \overset{\{\tau_1\}}{F}{}^{i_{\rho_1}}_{j_{\sigma_1}}\ \overset{\{\tau_2\}}{F}{}^{i_{\rho_2}}_{j_{\sigma_2}}\ \dots\ \overset{\{\tau_t\}}{F}{}^{i_{\rho_t}}_{j_{\sigma_t}};\ \overset{\{0\}}{F}{}^i_j = \delta^i_j\ \overset{\{1\}}{F}{}^i_j = F^i_j$$

$$\bigoplus = \begin{cases} \rho_1, \rho_2, \dots, \rho_t \in \overline{1,p} & (\rho_1 < \rho_2 < \dots < \rho_t), \\ \sigma_1, \sigma_2, \dots, \sigma_t \in \overline{1,q} & (\sigma_i \text{ are mutually different}), \\ \tau_1, \tau_2, \dots, \tau_t \in 0, 1; & \end{cases}$$

$$\star = \{\tau_1, \tau_2, \dots, \tau_t\}, \qquad \diamond = \left\{ \begin{array}{c} \rho_1, \rho_2, \dots, \rho_t \\ \sigma_1, \sigma_2, \dots, \sigma_t \\ \tau_1, \tau_2, \dots, \tau_t \end{array} \right\}.$$

2 Problem of general trace decomposition for spaces with quaternionic structure

Now we will generalize the trace decomposition problem and F-decomposition problem for spaces with quaternionic structure. The geometry of these spaces was studied for example in [1], [2] [3].

Let $\overset{\{1\}}{F}{}^i_j$, $\overset{\{2\}}{F}{}^i_j$ are affinors fulfilling the next relations.

$$\overset{\{1\}}{F}{}^i_\alpha\ \overset{\{1\}}{F}{}^\alpha_j = -\delta^i_j; \quad \overset{\{2\}}{F}{}^i_\alpha\ \overset{\{2\}}{F}{}^\alpha_j = -\delta^i_j; \quad \overset{\{1\}}{F}{}^i_\alpha\ \overset{\{2\}}{F}{}^\alpha_j + \overset{\{2\}}{F}{}^i_\alpha\ \overset{\{1\}}{F}{}^\alpha_j = 0 \tag{2}$$

Denote

$$\overset{\{0\}}{F}{}^i_j = \delta^i_j; \quad \overset{\{3\}}{F}{}^i_j = \overset{\{1\}}{F}{}^i_\alpha\ \overset{\{2\}}{F}{}^\alpha_j = -\overset{\{2\}}{F}{}^i_\alpha\ \overset{\{1\}}{F}{}^\alpha_j \tag{3}$$

A space has a *quaternionic structure* if conditions (2) and (3) hold.
A tensor $T\ \in E^p_q$ is *quaternionic traceless* if the next equation is true.

$$\forall k \in \overline{1,p}; \qquad \forall r \in \overline{1,q}: \qquad T^{\dots i_{k-1} \alpha i_{k+1} \dots}_{\dots j_{r-1} \beta j_{r+1} \dots}\ \overset{\{\tau_i\}}{F}{}^\beta_\alpha = 0 \quad \text{where } \tau_i \in \overline{0;3} \tag{4}$$

3 A general trace decomposition of tensor in spaces with quaternionic structure

Theorem 1 and Theorem 2 can be generalized for spaces with quaternionic structure.

Theorem 3 *Let T be a tensor of type (p,q). If $n > 4(p+q-2)$ then there exist a unique decomposition of T in the form*

$$T^{i_1 i_2 \dots\, i_p}_{j_1 j_2 \dots\, j_q} = B^{i_1 i_2 \dots\, i_p}_{j_1 j_2 \dots\, j_q} + \sum_{t=1}^{min\{p,q\}} \sum_{\bigoplus} \overset{\star}{Q}{}^{i_{\rho_1} i_{\rho_2} \dots\, i_{\rho_t}}_{j_{\sigma_1} j_{\sigma_2} \dots\, j_{\sigma_t}} \overset{\diamond}{B}{}^{\dots}_{\dots} \tag{5}$$

where

$$\bigoplus = \begin{cases} \rho_1, \rho_2, \dots\ \rho_t \in \overline{1,p} & (\rho_1 < \rho_2 < \dots\ < \rho_t) \\ \sigma_1, \sigma_2, \dots\ \sigma_t \in \overline{1,q} & (\sigma_i \quad \text{are mutually different}) \\ \tau_1, \tau_2, \dots\ \tau_t \in 0;1;2;3 & \end{cases} \tag{6}$$

$$\star = \{\tau_1, \tau_2 \dots\ \tau_t\}; \qquad \diamond = \left\{ \begin{array}{c} \rho_1, \rho_2 \dots\ \rho_t \\ \sigma_1, \sigma_2, \dots\ \sigma_t \\ \tau_1, \tau_2, \dots\ \tau_t \end{array} \right\}.$$

The tensors $B^{\dots}_{\dots}$ and $\overset{\diamond}{B}$ are quaternionic traceless and

$$\overset{\star}{Q}{}^{i_{\rho_1} i_{\rho_2} \dots\, i_{\rho_t}}_{j_{\sigma_1} j_{\sigma_2} \dots\, j_{\sigma_t}} \equiv \overset{\{\tau_1\}}{F}{}^{i_{\rho_1}}_{j_{\sigma_1}} \overset{\{\tau_2\}}{F}{}^{i_{\rho_2}}_{j_{\sigma_2}} \dots \overset{\{\tau_t\}}{F}{}^{i_{\rho_t}}_{j_{\sigma_t}},$$

where $\overset{\{\tau_i\}}{F}{}^{i}_{j}$ is the structure fulfilling (3).

Proof: Let $min\{p,q\} = p$. Then the relationship (5) can be rewritten in the next form.

$$T^{i_1 i_2 \dots i_p}_{j_1 j_2 \dots j_q} = B^{i_1 i_2 \dots i_p}_{j_1 j_2 \dots j_q} + \sum_{t=1}^{p} \sum_{\bigoplus} \overset{\star}{Q}{}^{i_{\rho_1} i_{\rho_2} \dots i_{\rho_t}}_{j_{\sigma_1} j_{\sigma_2} \dots j_{\sigma_t}} \overset{\diamond}{B}{}^{\dots}_{\dots}, \tag{7}$$

where condition (6) again holds for indices $\star$, $\diamond$, $\rho_1, \rho_2, \dots, \rho_t, \sigma_1, \sigma_2, \dots, \sigma_t$.
We need some relations to prove the theorem. Let for any $t \in \overline{1,p}$

$$V_1 \equiv n + 4(p+q-2) \quad ; V_t \equiv nV_{t-1} + 4(p+q-2) \quad \text{for } t \geq 2\ . \tag{8}$$

Then it is easy to prove that for $n > 4(p+q-2)$ the inequality

$$n^t > V_t - n^t \tag{9}$$

holds.

I. We assume that the tensor T has a decomposition of the form (7) where the tensors $B^{i_1 i_2 \dots i_p}_{j_1 j_2 \dots j_q}$ and $\overset{\diamond}{B}$ are traceless. If we contract (7) with $\overset{\star}{Q}{}^{j_1 j_2 \dots j_p}_{i_1 i_2 \dots i_p}$ we obtain the next equation.

$$n^p \overset{\square}{B}_{\dots} + \sum \overset{\triangle}{B}_{\dots} = \bar{T}^{s_1 s_2 \dots s_p}_{\dots s_1 s_2 \dots s_p \dots}, \tag{10}$$

where $\bar{T}^{s_1 s_2 \dots s_p}_{\dots s_1 s_2 \dots s_p \dots}$ is a contraction of the tensor T according to all p contravariant indices and

$$(11)\qquad \square = \left\{ \begin{array}{c} 1;\ 2;\ \dots;\ p \\ \sigma_1;\sigma_2;\dots;\sigma_p \\ \tau_1;\tau_2;\dots;\tau_t \end{array} \right\}; \quad \triangle = \left\{ \begin{array}{c} 1;\ 2;\ \dots;\ p \\ \eta_1;\eta_2;\dots;\eta_p \\ \omega_1;\omega_2;\dots;\omega_p \end{array} \right\},$$

where

$$(12)\qquad \begin{array}{c} \{\eta_1;\eta_2;\dots;\eta_p\} \neq \{\sigma_1;\sigma_2;\dots;\sigma_p\} \quad \eta_i \in \overline{1,q}\ , \\ \text{and } \{\tau_1;\tau_2;\dots;\tau_p\} \neq \{\omega_1;\omega_2;\dots;\omega_p\} \quad \omega_i \in \overline{0;3}\ , \end{array}$$

(10) is a system of (inhomogenous) linear equations for the unknowns

$$(13)\qquad \overset{\square}{B}_{\dots}\ .$$

If $n^p > V_p - n^p$, the system (10) has a unique solution and for any tensor T, all tensors (13) are uniquely determined.

II. Now we compute in a similar way the tensors $\overset{\bar{\triangle}}{B}{}^s_{\dots}$, where

$$(14)\qquad \bar{\triangle} = \left\{ \begin{array}{c} \rho_1;\rho_2;\dots;\rho_{p-1} \\ \sigma_1;\sigma_2;\dots;\sigma_{p-1} \\ \tau_1;\tau_2;\dots;\tau_{p-1} \end{array} \right\}.$$

Suppose that the tensors $B^{i_1 i_2 \dots i_p}_{j_1 j_2 \dots j_q}$ and $\overset{\diamond}{B}{}^{\dots}_{\dots}$ $(t \le p-2)$ are traceless. Contracting (7) with $\overset{\star}{Q}{}^{j_{\sigma_1} j_{\sigma_2} \dots j_{\sigma_{p-1}}}_{i_{\rho_1} i_{\rho_2} \dots i_{\rho_{p-1}}}$ we obtain a system of equations

$$(15)\qquad n^{p-1}\, \overset{\bar{\triangle}}{B}{}^s_{\dots} + \sum \overset{\tilde{\triangle}}{B}{}^s_{\dots} = \dots\ ,$$

where

$$(16)\qquad \tilde{\triangle} = \left\{ \begin{array}{c} \eta_1;\eta_2;\dots;\eta_{p-1} \\ \mu_1;\mu_2;\dots;\mu_{p-1} \\ \omega_1;\omega_2;\dots;\omega_{p-1} \end{array} \right\}$$

and the right side of (15) is the contraction of the tensor T.

$$(17)\qquad \overset{\bar{\triangle}}{B}{}^s_{j_p \dots j_q}$$

are unknowns in the system (15) of linear equations. If $n^{p-1} > V_{p-1} - n^{p-1}$ this system has a unique solution. (It is true for $n > 4(p+q-2)$). Now we must verify the tracelessness of the tensors (17). If we contract (15) with $\overset{\{\omega\}}{F}{}^{j_\sigma}_{s}$ $\sigma \neq (\sigma_1;\sigma_2;\dots;\sigma_{p-1})$; $\omega = 0;1;2;3$, we get the system

$$n^{p-1}\, \overset{\bar{\triangle}}{B}{}^\alpha_{\dots\beta\dots}\, \overset{\{0\}}{F}{}^\beta_\alpha + \sum \overset{\tilde{\triangle}}{B}{}^\alpha_{\dots\beta\dots}\, \overset{\{0\}}{F}{}^\beta_\alpha + \sum \overset{\tilde{\triangle}}{B}{}^\alpha_{\dots\beta\dots}\, \overset{\{1\}}{F}{}^\beta_\alpha +$$
$$+ \sum \overset{\tilde{\triangle}}{B}{}^\alpha \dots \beta \dots\, \overset{\{2\}}{F}{}^\beta_\alpha + \sum \overset{\tilde{\triangle}}{B}{}^\alpha_{\dots\beta\dots}\, \overset{\{3\}}{F}{}^\beta_\alpha = 0\ ,$$

$$\begin{aligned}
&n^{p-1}\,\overset{\tilde\triangle}{B}{}^{\alpha}_{\dots\beta\dots}\,\overset{\{1\}}{F}{}^{\beta}_{\alpha}+\sum\overset{\tilde\triangle}{B}{}^{\alpha}_{\dots\beta\dots}\,\overset{\{0\}}{F}{}^{\beta}_{\alpha}+\sum\overset{\tilde\triangle}{B}{}^{\alpha}_{\dots\beta\dots}\,\overset{\{1\}}{F}{}^{\beta}_{\alpha}+\\
&\quad+\sum\overset{\tilde\triangle}{B}{}^{\alpha}_{\dots\beta\dots}\,\overset{\{2\}}{F}{}^{\beta}_{\alpha}+\sum\overset{\tilde\triangle}{B}{}^{\alpha}_{\dots\beta\dots}\,\overset{\{3\}}{F}{}^{\beta}_{\alpha}=0\\
&n^{p-1}\,\overset{\tilde\triangle}{B}{}^{\alpha}_{\dots\beta\dots}\,\overset{\{2\}}{F}{}^{\beta}_{\alpha}+\sum\overset{\tilde\triangle}{B}{}^{\alpha}_{\dots\beta\dots}\,\overset{\{0\}}{F}{}^{\beta}_{\alpha}+\sum\overset{\tilde\triangle}{B}{}^{\alpha}_{\dots\beta\dots}\,\overset{\{1\}}{F}{}^{\beta}_{\alpha}+\\
&\quad+\sum\overset{\tilde\triangle}{B}{}^{\alpha}_{\dots\beta\dots}\,\overset{\{2\}}{F}{}^{\beta}_{\alpha}+\sum\overset{\tilde\triangle}{B}{}^{\alpha}_{\dots\beta\dots}\,\overset{\{3\}}{F}{}^{\beta}_{\alpha}=0\\
&n^{p-1}\,\overset{\tilde\triangle}{B}{}^{\alpha}_{\dots\beta\dots}\,\overset{\{3\}}{F}{}^{\beta}_{\alpha}+\sum\overset{\tilde\triangle}{B}{}^{\alpha}_{\dots\beta\dots}\,\overset{\{0\}}{F}{}^{\beta}_{\alpha}+\sum\overset{\tilde\triangle}{B}{}^{\alpha}_{\dots\beta\dots}\,\overset{\{1\}}{F}{}^{\beta}_{\alpha}+\\
&\quad+\sum\overset{\tilde\triangle}{B}{}^{\alpha}_{\dots\beta\dots}\,\overset{\{2\}}{F}{}^{\beta}_{\alpha}+\sum\overset{\tilde\triangle}{B}{}^{\alpha}_{\dots\beta\dots}\,\overset{\{3\}}{F}{}^{\beta}_{\alpha}=0\ ,
\end{aligned}\tag{18}$$

which has only the trivial solution for $n>4(p+q-2)$.

III. We will continue in a similar way and we find the tensors

$$\overset{\overset{=}{\triangle}}{B}{}^{\dots}_{\dots}\qquad\overset{\overset{\equiv}{\triangle}}{B}{}^{\dots}_{\dots}\qquad\dots\qquad\overset{\substack{\rho_1\\ \sigma_1\\ \tau_1}}{B}{}^{\dots}_{\dots}\qquad B^{\dots}_{\dots}\tag{19}$$

where

$$\overset{=}{\triangle}=\left\{\begin{array}{c}\rho_1;\rho_2;\dots;\rho_{p-2}\\ \sigma_1;\sigma_2;\dots;\sigma_{p-2}\\ \tau_1;\tau_2;\dots;\tau_{p-2}\end{array}\right\}\quad,\quad\overset{\equiv}{\triangle}=\left\{\begin{array}{c}\rho_1;\rho_2;\dots;\rho_{p-3}\\ \sigma_1;\sigma_2;\dots;\sigma_{p-3}\\ \tau_1;\tau_2;\dots;\tau_{p-3}\end{array}\right\}\tag{20}$$

and we prove their tracelessness.

Let tensors $\overset{\diamond}{B}{}^{\dots}_{\dots}$ (where $\diamond$ fulfill again the condition (6)) be traceless and now we contract (7) with $\overset{\{\tau_1\}}{Q}{}^{j_{\sigma_1}}_{i_{\rho_1}}$. We get a system of linear equations

$$n\,\overset{\substack{\rho_1\\ \sigma_1\\ \tau_1}}{B}{}^{\dots}_{\dots}+\sum\overset{\substack{\eta\\ \mu\\ \omega}}{B}{}^{\dots}_{\dots}=\dots\qquad\eta\in\overline{1,p}\setminus\{\rho_1\};\quad\mu\in\overline{1,q}\setminus\{\sigma_1\}\omega\in\overline{0;3}\tag{21}$$

with the unknown tensors

$$\overset{\substack{\rho\\ \sigma\\ \tau}}{B}\quad,\quad\rho\in\overline{1,p};\ \sigma\in\overline{1,q}.\tag{22}$$

The right side of (21) are again determined by the tensor T . If $n>4(p+q-2)$ the system has exactly one solution.

Contracting (21) by the tensor $\overset{\{\omega\}}{F}{}^{j_{\sigma_1}}_{i_{\rho_1}}$ we get the system

$$\begin{aligned}
&n\,\overset{\substack{\rho\\ \sigma\\ \tau}}{B}{}^{\dots\alpha\dots}_{\dots\beta\dots}\,\overset{\{0\}}{F}{}^{\beta}_{\alpha}+\sum\overset{\substack{\eta\\ \mu\\ \omega}}{B}{}^{\dots\alpha\dots}_{\dots\beta\dots}\,\overset{\{0\}}{F}{}^{\beta}_{\alpha}+\sum\overset{\substack{\eta\\ \mu\\ \omega}}{B}{}^{\dots\alpha\dots}_{\dots\beta\dots}\,\overset{\{1\}}{F}{}^{\beta}_{\alpha}+\\
&\quad+\sum\overset{\substack{\eta\\ \mu\\ \omega}}{B}{}^{\dots\alpha\dots}_{\dots\beta\dots}\,\overset{\{2\}}{F}{}^{\beta}_{\alpha}+\sum\overset{\substack{\eta\\ \mu\\ \omega}}{B}{}^{\dots\alpha\dots}_{\dots\beta\dots}\,\overset{\{3\}}{F}{}^{\beta}_{\alpha}=0\ ,\\
&n\,\overset{\substack{\rho\\ \sigma\\ \tau}}{B}{}^{\dots\alpha\dots}_{\dots\beta\dots}\,\overset{\{1\}}{F}{}^{\beta}_{\alpha}+\sum\overset{\substack{\eta\\ \mu\\ \omega}}{B}{}^{\dots\alpha\dots}_{\dots\beta\dots}\,\overset{\{0\}}{F}{}^{\beta}_{\alpha}+\sum\overset{\substack{\eta\\ \mu\\ \omega}}{B}{}^{\dots\alpha\dots}_{\dots\beta\dots}\,\overset{\{1\}}{F}{}^{\beta}_{\alpha}+
\end{aligned}$$

$$+\sum \overset{\eta}{\underset{\omega}{\overset{\mu}{B}}}{}^{\dots\alpha\dots}_{\dots\beta\dots}\ \overset{\{2\}}{F}{}^{\beta}_{\alpha} + \sum \overset{\eta}{\underset{\omega}{\overset{\mu}{B}}}{}^{\dots\alpha\dots}_{\dots\beta\dots}\ \overset{\{3\}}{F}{}^{\beta}_{\alpha} = 0\ ,$$

$$n\ \overset{\rho}{\underset{\tau}{\overset{\sigma}{B}}}{}^{\dots\alpha\dots}_{\dots\beta\dots}\ \overset{\{2\}}{F}{}^{\beta}_{\alpha} + \sum \overset{\eta}{\underset{\omega}{\overset{\mu}{B}}}{}^{\dots\alpha\dots}_{\dots\beta\dots}\ \overset{\{0\}}{F}{}^{\beta}_{\alpha} + \sum \overset{\eta}{\underset{\omega}{\overset{\mu}{B}}}{}^{\dots\alpha\dots}_{\dots\beta\dots}\ \overset{\{1\}}{F}{}^{\beta}_{\alpha} +$$

$$+\sum \overset{\eta}{\underset{\omega}{\overset{\mu}{B}}}{}^{\dots\alpha\dots}_{\dots\beta\dots}\ \overset{\{2\}}{F}{}^{\beta}_{\alpha} + \sum \overset{\eta}{\underset{\omega}{\overset{\mu}{B}}}{}^{\dots\alpha\dots}_{\dots\beta\dots}\ \overset{\{3\}}{F}{}^{\beta}_{\alpha} = 0\ ,$$

$$n\ \overset{\rho}{\underset{\tau}{\overset{\sigma}{B}}}{}^{\dots\alpha\dots}_{\dots\beta\dots}\ \overset{\{3\}}{F}{}^{\beta}_{\alpha} + \sum \overset{\eta}{\underset{\omega}{\overset{\mu}{B}}}{}^{\dots\alpha\dots}_{\dots\beta\dots}\ \overset{\{0\}}{F}{}^{\beta}_{\alpha} + \sum \overset{\eta}{\underset{\omega}{\overset{\mu}{B}}}{}^{\dots\alpha\dots}_{\dots\beta\dots}\ \overset{\{1\}}{F}{}^{\beta}_{\alpha} +$$

$$+\sum \overset{\eta}{\underset{\omega}{\overset{\mu}{B}}}{}^{\dots\alpha\dots}_{\dots\beta\dots}\ \overset{\{2\}}{F}{}^{\beta}_{\alpha} + \sum \overset{\eta}{\underset{\omega}{\overset{\mu}{B}}}{}^{\dots\alpha\dots}_{\dots\beta\dots}\ \overset{\{3\}}{F}{}^{\beta}_{\alpha} = 0\ , \tag{23}$$

which has again only the trivial solution for $n > 4(p+q-2)$. Thus the tensors $\overset{\rho}{\underset{\tau}{\overset{\sigma}{B}}}{}^{\dots}_{\dots}$ are traceless.

The tensor $B^{\dots}_{\dots}$ now can be computed from (7) because all tensors $\overset{\diamond}{B}{}^{\dots}_{\dots}$ are determined. If we combine the contraction of (7) with the tensor $\overset{\{\omega\}}{F}{}^{j_{\sigma_1}}_{i_{\rho_1}}$ and previous calculation we can prove the tracelessness of the tensor $B^{\dots}_{\dots}$ and the proof is finished.

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