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# On Hermite Interpolation Formula

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Let us consider an arbitrary finite interval $[a,b]$ on the real line, $x_0, x_1, \ldots, x_n$ distinct knots in this interval, with the multiplicities $\alpha_0, \alpha_1, \ldots, \alpha_n$, respectively, where $\alpha = (\alpha_1, \ldots, \alpha_n)$ is a sequence of positive integers, and

$$D^{\alpha}[a,b] = \{f : [a,b] \to \mathbf{R}\ ;\ \exists f^{(\alpha_k - 1)}(x_k), k = \overline{0,n}\}.$$

Let $N = \sum\limits_{k=0}^{n} \alpha_k - 1$ and $\Pi_N$ be the linear space of all real polynomials of the degree $\leq N$. We denote by $H : D^{\alpha}[a,b] \to \Pi_N$ the Hermite interpolation operator defined by the conditions

$$(1) \qquad (Hf)^{(\nu_k)}(x_k) = f^{(\nu_k)}(x_k), k = \overline{0,n},\ \nu_k = \overline{0, \alpha_k - 1}.$$

Further we consider the set

$$I_p = \{(i_0, \mu_{i_0}), (i_1, \mu_{i_1}), \ldots, (i_p, \mu_{i_p})\} \quad , \quad \tau = \{i_0, i_1, ..., i_p\}$$

where $p \in \{0, 1, \ldots, n\}$ is fixed, $i_j \in \{0, 1, ..., n\}$, $j = \overline{0, p}$, are distinct and fixed numbers and $\mu_{i_j} \in \{0, \ldots, \alpha_{i_j} - 1\}, j = \overline{0, p}$, are fixed numbers.

Let $y^*_{i_0, \mu_{i_0}}$, $y^*_{i_1, \mu_{i_1}}, \ldots$, $y^*_{i_p, \mu_{i_p}} \in \mathbf{R}$ fixed numbers such that

$$|y^*_{i_j, \mu_{i_j}} - f^{(\mu_{i_j})}(x_{i_j})| < \varepsilon_{i_j}, j = \overline{0, p},$$

where $\varepsilon_{i_j} > 0$, $j = 0, p$, are known.

We denote by $(H^* f)(x)$ Hermite$'$s interpolating polynomial defined by the conditions

$$(H^* f)^{(\nu_k)}(x_k) = \begin{cases} f^{(\nu_k)}(x_k) & \text{,if} \quad (k, \nu_k) \in \overline{0, n} \times \overline{0, \alpha_k - 1} \\ & \quad (k, \nu_k) \neq (i_j, \mu_{i_j}), j = \overline{0, p} \\ y^*_{i_j, \mu_{i_j}} & \text{,if} \quad (k, \nu_k) \in \overline{0, n} \times \overline{0, \alpha_k - 1} \\ & \quad (k, \nu_k) = (i_j, \mu_{i_j}), j = \overline{0, p}. \end{cases}$$

(2)

Further let us consider that $Rf$ and $R^* f$ are the remainders in Hermite $'$s interpolation formulas coresponding to conditions (1), respectively (2). The following problem may be formulated:

**Problem.** *If $f \in C^{N+1}[a, b]$ , find upper bounds for the*

$$|(Rf)(x) - (R^* f)(x)| \quad , \quad x \in [a, b]$$

*only in terms of*

$$\omega(x) \ := \prod_{k=0, k \notin \tau}^{n} (x - x_k) \ \ , \ \ \Omega_\tau(x) \ := \prod_{k \in \tau} (x - x_k) \ ,$$

$$\epsilon(q) \ := (\frac{1}{q+1} \sum_{j=0}^{q} \varepsilon_{i_j}^{q})^{\frac{1}{q}} \quad , \quad \| f^{(N+1)} \| \quad ,$$

$q$ *being fixed (positive) numbers and* $\| \cdot \|$ *the uniform norm on* $[a, b]$ .